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Modulation instability and ion-acoustic rogue waves in a strongly coupled collisional plasma with nonthermal nonextensive electrons

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Abstract
The nonlinear propagation of ion-acoustic waves is theoretically reported in a collisional plasma containing strongly coupled ions and nonthermal electrons featuring Tsallis distribution. For this purpose, the nonlinear integro-differential form of the generalized hydrodynamic model is used to investigate the strong-coupling effect. The modified complex Ginzburg–Landau equation with a linear dissipative term is derived for the potential wave amplitude in the hydrodynamic regime, and the modulation instability of ion-acoustic waves is examined. When the dissipative effect is neglected, the modified complex Ginzburg–Landau equation reduces to the nonlinear Schrödinger equation. Within the unstable region, two different types of second-order ion-acoustic rogue waves including single peak type and rogue wave triplets are discussed. The effect of the plasma parameters on the rogue waves is also presented.

Keywords: ion-acoustic rogue waves, strongly coupled plasma, generalized hydrodynamic model, modulation instability

(Some figures may appear in colour only in the online journal)

I. Introduction

In recent years, the study of nonlinear wave propagation in strongly coupled plasmas has seen an explosive growth because of the potential diverse applications in laboratory environments, compact astrophysical objects, as well as condensed matter systems. In theory, the formation of strongly coupled plasmas was firstly predicted by Ikezi \cite{1}. The average Coulomb interaction energy between neighboring ions can be in the order of or exceed the thermal energy, which makes them the strongly coupled system \cite{2}. The degree of strong coupling between ions is traditionally characterized by the parameter $\Gamma_i = Z_i^2 e^2/(k_B T_i d)$ \cite{3}. Here, $Z_i$ is the ion charge state, $e$ the fundamental charge, $k_B$ the Boltzmann constant, $T_i$ the ion temperature, and $d$ the mean inter-particle distance (Wigner-Seitz cell radius). For $\Gamma_i < 1$, the plasma represents a gaseous medium; then it changes to the liquid-like state with density growth or with cooling when $\Gamma_i \geq 10$. With a further enhancement of $\Gamma_i$, the ions organize themselves into the crystal structure in the range $170 \leq \Gamma_i \leq 180$; in the regime $\Gamma_i > 180$, the Coulomb force completely dominates over the system and the ions arrange themselves in a periodic lattice structure. In the latter case, the plasma behaves like a solid...
with long-range order and supports various lattice modes. Because of the presence of strongly coupled Coulomb mixtures of ions, the properties of such plasma have attracted much attention recently [4, 5].

In addition, theoretical and experimental studies have found that strong coupling between ions significantly modifies the collective behavior named ‘ion-acoustic waves’. In [6–11], the theoretical treatments of ion-acoustic waves in strongly coupled plasmas have been discussed by many researchers. In laboratory, the experimental excitations of ion-acoustic waves, taking into account the influence of strongly coupled correlation, have been reported recently [12, 13].

One interesting research area for finite amplitude ion-acoustic waves is modulation instability (MI), which may occur because of the interaction between high- and low-frequency modes, parametric wave coupling, or the nonlinear self-interaction of carrier waves. By incorporating the dissipative effect, the plasma is no longer Hamiltonian. A typical paradigm of an evolution equation, describing the slow modulation of a periodic pattern in space and time near the threshold, is the complex Ginzburg–Landau (CGL) equation. If dissipation is neglected, the CGL equation can be reduced to the nonlinear Schrödinger (NLS) equation which possesses the characteristics of an integrable system (a subclass of the Hamiltonian system).

In the MI region, the NLS equation admits the interesting rational solutions named ‘rogue waves’ [14] (also known as ‘freak waves’, ‘extreme waves’, or ‘killer waves’). Recently, this kind of nonlinear doubly-localized structure has been a hot topic in plasma physics. The creation of rogue waves is mainly because of MI: a small perturbation of a plane wave could lead to the exponential growth of the wave with the possibility of increasing up to very high amplitude, and then to a decay so that it finally ‘disappears without a trace’ [15–20]. Rogue waves have been theoretically investigated in many plasma systems [10, 21–24] and confirmed in laboratory experiments [25, 26].

In this work, we present the nonlinear theory of small amplitude ion-acoustic structure in a dissipative plasma, taking into account the effect of strong coupling between ions, collision between ions and neutral particles, effective ion temperature, nonthermal electrons featuring Tsallis distribution, etc. Considering various effects can give a much more complete picture in the plasma system.

The paper is organized as follows. In section II, we present the generalized hydrodynamic model for our plasma system with strongly correlated ions and derive a set of simplified basic equations. In section III, the reductive perturbation method is applied to derive the modified CGL equation with a linear dissipative term for the potential wave amplitude. In section IV, we discuss the MI of ion-acoustic waves in the strongly coupled regime. When the dissipative effect vanishes, the modified CGL equation reduces to the standard NLS equation. By using the NLS equation, we investigate the nonlinear evolution of second-order ion-acoustic rogue waves with two free parameters. Finally, conclusions are given in section V.

II. Theoretical model and basic equations

We investigate the nonlinear propagation of ion-acoustic waves in an unmagnetized collisional plasma whose components are strongly coupled ions and electrons. Such two-component strongly coupled plasma can be found in laboratory experiments. For example, Castro et al [12] experimentally investigated the ion-acoustic waves in ultracold neutral plasma whose constituents were strongly coupled ions and electrons. In this experiment, the ultracold neutral plasma with controlled density perturbations was created by photoionization of laser-cooled strontium atoms from a magneto-optical trap. Later, a similar scheme was used to create two-component (electrons and strong-coupling ions) ultracold neutral plasmas in the strongly coupled regime and the ion-acoustic waves were observed through density and velocity perturbations [13].

At equilibrium we have \( n_{e0} = Zn_{i0} \), where \( n_{e0} (n_{i0}) \) is the equilibrium ion (electron) number density. Here, we do not consider the electron inertia because the electron thermal speed is much larger than that of ions. Since the strongly coupled plasma can be created in a state that is far from the thermal equilibrium, we will use the following nonextensive nonthermal distribution function [27] for electrons:

\[
n_{i} = n_{e0}\left[1 + (q - 1)\left(\frac{\varphi}{T_{e}}\right)^{\frac{q+1}{q-1}}\left(1 + \rho_{1}\left(\frac{\varphi}{T_{e}}\right)\right) + \rho_{2}\left(\frac{\varphi}{T_{e}}\right)^{2}\right],
\]

(1)

where \( T_{e} \) is the electron temperature, \( \varphi \) the electrostatic scalar potential, \( \rho_{1} = -16qa(3 - 14q + 15q^{2} + 2a) \), \( \rho_{2} = 16(2q - 1)qa(3 - 14q + 15q^{2} + 2a) \). The parameter \( a \) determines the number of nonthermal electrons in our model, and \( q \) denotes the degree of nonextensivity. When \( a = 0 \), the density (1) reduces to the \( q \)-nonextensive electron density [28]. When \( q \rightarrow 1 \), the density (1) reduces to the well-known nonthermal electron density proposed by Cairns et al [29].

To describe the dynamics of our plasma system, we employ the so-called ‘generalized hydrodynamic’ model to investigate the strong-coupling effect between ions. By introducing the viscoelastic coefficients into the hydrodynamic equations, the generalized hydrodynamic model has been successfully applied to a number of strongly coupled media [30, 31]. Because of the ion-ion coupling effect, the modified momentum equation in the generalized hydrodynamic model takes the following integro-differential form

\[
\frac{\partial u_{i}}{\partial t} + u_{i} \frac{\partial u_{i}}{\partial x} + \frac{Z_{e}e}{m_{i}} \frac{\partial \varphi}{\partial x} + \frac{T_{e}}{m_{i}} \frac{\partial n_{i}}{\partial x} + \nu u_{i} = \int_{-\infty}^{t'} dt' \int_{-\infty}^{t+\infty} dx' \eta_{i}\left(x - x', t - t'\right) u_{i}(x', t'),
\]

(2)

where \( m_{i} \), \( n_{i} \), and \( u_{i} \) represent the ion mass, number density, ion fluid velocity, and ion-neutral particle collision frequency, respectively. Furthermore, \( T_{e} = (T_{b} + \mu_{i}T_{i}) \) is the effective
ion temperature to be explained later, \( T_i \) is the ion temperature. The function \( \eta_i \) is a non-local viscoelastic operator which accounts for the non-local and memory effects.

Let the symbol \( Z(x, t) \) stand for the left-hand side of equation (2), and take the Fourier transform for the variable \( x \). Then, we obtain

\[
Z(\theta, t) = \int_{-\infty}^{t} \exp \left( -\frac{t - t'}{\tau(\theta)} \right) \tilde{\eta}_i(\theta, t - t') \tilde{u}_i(\theta, t'),
\]

(3)

where \( \theta \) is the Fourier transform variable for space. A model expression for the viscoelastic function, chosen to emphasize the separate roles of the generalized viscosity and the relaxation time, is given by [32]

\[
\tilde{\eta}_i(\theta, t) = \frac{\bar{\eta}_i(\theta)}{\tau(\theta)} \exp \left( -\frac{t - t'}{\tau(\theta)} \right)
\]

(4)

Here, \( \bar{\eta}_i(\theta) \) and \( \tau(\theta) \) are the generalized viscosity term and relaxation time, respectively. The above memory function, derived by making the Markov approximation for the evolution of the memory function, can provide a good description of the collective behavior of strongly coupled systems for both high- and low-frequency limits and for all wavelengths.

Taking the partial time derivative of equation (3) and considering

\[
\frac{\partial \bar{\eta}_i(\theta, t)}{\partial t} = -\frac{\tilde{\eta}_i(\theta, 0)}{\tau(\theta)} \bar{u}_i(\theta, t)
\]

and

\[
-\int_{-\infty}^{t} \exp \left( -\frac{t - t'}{\tau(\theta)} \right) \tilde{\eta}_i(\theta, t - t') \tilde{u}_i(\theta, t').
\]

(5)

Via the relationship equation (3)+\( \tau(\theta) \) equation (5) and the equation \( \bar{\eta}_i(\theta, 0) = \frac{\bar{\eta}_i(0)}{\tau(0)} \), we obtain the following general form of the nonlinear momentum equation

\[
\left( 1 + \tau(\theta) \frac{\partial}{\partial t} \right) Z(\theta, t) = \eta_i(\theta) \tilde{u}_i(\theta, t).
\]

(6)

Here, the second term on the left-hand side of the above equation appears because of the strong-coupling effect between ions. In the present treatment, we relax the relaxation time \( \tau(\theta) = \tau_m \) for simplicity. The viscosity term \( \eta_i(\theta) \) can be taken as the following form

\[
\eta_i(\theta) = \frac{(b + \frac{4}{3}s)^2}{m_i n_i 0},
\]

(7)

where \( b \) and \( s \) are the transport coefficients of bulk and shear viscosities, respectively.

Considering the expression (7) and assumption \( n_i \approx n_0 \), and taking the inverse Fourier transform of equation (6), we have the following generalized momentum equation

\[
\left( 1 + \tau_m \frac{\partial}{\partial t} \right) \left( m_i \frac{\partial u_i}{\partial t} + \frac{\partial n_i}{\partial x} + n_i Z e \frac{\partial \varphi}{\partial x} \right) + T_{\text{eff}} \frac{\partial n_i}{\partial x} + \nu m_i u_i = \eta_i \frac{\partial u_i^2}{\partial x}.
\]

(8)

Here, \( \eta_i^* = (b + \frac{4}{3}s)^2 \). There are various approaches for calculating the transport coefficients in the above ion momentum equation, which has been widely discussed in [33–38]. The coefficients \( b, s, \tau_m, \) and \( T_{\text{eff}} \) are the functions of the ion Coulomb coupling parameter \( \Gamma_i \), which introduces the strongly coupled effects into the collective behavior of our plasma model. It has been indicated that the viscosity coefficient \( (b + \frac{4}{3}s)n_0,k_BT_i \) has wide minima \( \sim 1 \) for \( 1 < \Gamma_i < 10 \); then it tends to increase as \( \Gamma_i^{4/3} \) for \( \Gamma_i > 10 \) [34]. The relaxation time \( \tau_m \) could be modeled as [35]

\[
\tau_m = \frac{(b + \frac{4}{3}s)}{n_0,k_BT_i^4} \left( 1 - \mu_i + \frac{4}{15} u(\Gamma_i) \right)^{-1},
\]

where \( u(\Gamma_i) \) is a measure of the excess internal ion energy. Clearly, the relaxation time \( \tau_m \) becomes high in the strong coupling regime. The compressibility parameter \( \mu_i \), appearing in the above equation is given by [35, 38]

\[
\mu_i = \frac{1}{k_BT_i} \frac{\partial \mu_i}{\partial T_i} = \frac{1}{1 + \frac{1}{9} u(\Gamma_i) + \frac{\Gamma_i}{9} \left( \frac{1}{u(\Gamma_i)} - 1 \right)}.
\]

(9)

The expression for \( u(\Gamma_i) \) can be given as [33]

\[
u(\Gamma_i) = \begin{cases} 
-\frac{\sqrt{3}}{2} \Gamma_i^{3/2} , & \Gamma_i < 1, \\
-0.90 \Gamma_i + 0.95 \Gamma_i^{1/4} , & 1 \leq \Gamma_i \leq 160, \\
+0.18 \Gamma_i^{-1/4} - 0.80 , & 160 < \Gamma_i \leq 300, \\
1.5 - 0.90 \Gamma_i + 2980 \Gamma_i^{-2} , & 300 < \Gamma_i \leq 1000, \\
+840 \Gamma_i^{-1} + 1.1 \times 10^5 \Gamma_i^{-3} , & 1000 < \Gamma_i \leq 2000.
\end{cases}
\]

The effective ion temperature \( T_{\text{eff}} = (T_i + \mu_i T_i) \) contains two parts: one \( (T_i) \) is caused by electrostatic interaction between strongly coupled ions, the other \( (\mu_i T_i) \) is caused by the ion thermal pressure. In addition, the expression for \( T_i \) is given by [36, 37]

\[
T_i = \frac{N_{\text{in}}}{3} \frac{n_i}{Z_i} T_i (1 + \kappa) e^{-\kappa},
\]

where \( N_{\text{in}} \) is the number of the nearest neighbors that determine the plasma’s structure, \( \kappa = d/\lambda_{\text{Debye}} \) measures the screening of the ion charge by the plasma over a distance of the ion Debye length \( \lambda_{\text{Debye}} \).

Next, we normalize the physical quantities as follows:

\[
x \rightarrow x/l_0, \quad t \rightarrow t/\omega_i^{-1}, \quad n_i \rightarrow n_i/l_{n_0}, \quad u_i \rightarrow u_i/C_i, \quad \varphi \rightarrow e\varphi/C_i T_i, \quad \Gamma_i \rightarrow C_i T_i
\]

(9)

where \( Z_i = \sqrt{Z_i T_i/m_i}, \quad \omega_i^{-1} = \sqrt{m_i/(4\pi Z_i^2 e^2 n_0)}, \quad \kappa = \sqrt{T_i(4\pi n_0 Z_i e^2)}, \quad \lambda_{\text{Debye}} = \sqrt{T_i(4\pi n_0 Z_i e^2)} \). Thus, the normalized equations governing the non-linear dynamics of ion-acoustic waves in our strongly coupled plasma system are

\[
\frac{\partial u_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0,
\]

(10)
\[
\left\{1 + \frac{\pi_m}{\partial} \right\} \left[ n_l \left( \frac{\partial u}{\partial t} + u_l \frac{\partial u}{\partial x} + n_l \frac{\partial \varphi}{\partial x} \right) + \sigma \frac{\partial u}{\partial x} + \nu \lambda u_l \right] = \eta^2 \frac{\partial^2 u}{\partial x^2},
\]
\[
\frac{\partial^2 \varphi}{\partial x^2} = n_e - n_i. \tag{11}
\]

In the above equations, \( \sigma = T_e/T_i \) is the effective temperature ratio, \( \nu = \nu_I/\omega_i \), \( \eta^2 = \eta^2/(m_i \nu_i \omega_i \lambda^2) \). The normalized electron density is
\[
n_e = (1 + (q - 1) \varphi \omega^3) \times (1 + \rho_1 \varphi + \rho_2 \varphi^2). \tag{13}
\]

### III. Complex Ginzburg–Landau equation

To investigate the nonlinear wave modulation of ion-acoustic waves described by equations (10)–(12) in the hydrodynamic regime (\( \omega_i \mu \ll 1 \)), we introduce the slow space and time scales through the stretched variables:
\[
\xi = \epsilon (x - V \tau), \quad \tau = \epsilon^2 t,
\]
where \( \epsilon \) is a small expansion parameter, \( V \) is the wave group velocity along the propagation direction. The dependent variables \( n_i, u_i \) and \( \varphi \) are expanded as
\[
n_l = 1 + \sum_{m=1}^{\infty} \sum_{l=0}^{\infty} n_l^{(m)}(\xi, \tau) e^{i(kl - \omega t)},
\]
\[
u_l = \sum_{m=1}^{\infty} \sum_{l=0}^{\infty} \nu_l^{(m)}(\xi, \tau) e^{i(kl - \omega t)},
\]
\[
\varphi = \sum_{m=1}^{\infty} \sum_{l=0}^{\infty} \varphi_l^{(m)}(\xi, \tau) e^{i(kl - \omega t)}.
\]

Here, \( i = \sqrt{-1} \), \( k \) and \( \omega \) are real variables representing the fundamental wave number and angular frequency, respectively. Since \( n_i, u_i \) and \( \varphi \) must be real, all elements in equation (15) satisfy the reality condition \( A_l^{(l)} = A_l^{(l)^*} \) \( (A = n_i, u_i, \varphi) \), and the asterisk denotes complex conjugate.

Substitution of equations (14) and (15) into equations (10)–(12) yields the first order \( (m = 1) \) equations with \( l = 1 \):
\[
n_l^{(1)} = A_k \varphi_l^{(1)} + \omega \varphi_l^{(1)}, \quad u_l^{(1)} = A_l \varphi_l^{(1)}, \tag{16}
\]
with \( \omega \) satisfying the dispersion relation:
\[
\omega^2 + i \eta^2 k^2 \omega - \left(1 + c_i \sigma + k^2 \sigma \right) k^2 = 0. \tag{17}
\]

Here, \( c_i = [(q + 1) + \rho_1]/2 \), the coefficient \( A \) is given in the appendix. Clearly, we can see from equation (17) that the solutions of the dispersion relation in strongly coupled plasma are, generally speaking, complex. The complex wave frequency \( \omega = Re(\omega) + i Im(\omega) \) can be regarded as a function of the real wave number \( k \). In physics, the real part \( Re(\omega) \) of the complex wave frequency represents the ion-acoustic wave frequency, while the imaginary frequency \( Im(\omega) \) implies the damping rate.

Using the quantities (16), the second order \( (m = 2) \) equations with \( l = 1 \) are
\[
n_l^{(2)} = \left( -B_l - i A_l \omega + i A_l V (k) \right) \partial \varphi_l^{(1)} - \omega \varphi_l^{(1)},
\]
\[
u_l^{(2)} = \left( -A_l \omega + A_l V (k) \right) \partial \varphi_l^{(1)} - \omega \varphi_l^{(1)}, \tag{18}
\]
\[
\varphi_l^{(2)} = \left( -A_l \omega + A_l V (k) \right) \partial \varphi_l^{(1)} - \omega \varphi_l^{(1)}, \tag{19}
\]
with the compatibility condition
\[
V_g = \frac{\partial \omega}{\partial k} = -k(i \eta^2 k^2 + 2\omega). \tag{20}
\]

Recall that the above condition is the group velocity. Here, \( B \) and \( C \) are given in the appendix, \( \Xi_t \) is given by
\[
\Xi_t = -2\omega^2 + 2\sigma^2 k^2 - 4\sigma^2 k^2 \omega^2 - 4\sigma^2 k^4 \eta^2 + 2\omega^4 + 4i \omega k^2 \eta^2 k^2 - 2\omega^2 \eta^2 k^4.
\]

It is obvious from equation (20) that the group velocity is also complex for a strongly coupled plasma, i.e. \( V_g = Re(V_g) + i Im(V_g) \). It has been found that the real part \( Re(V_g) \) of group velocity indicates the physical group velocity [39], while the imaginary part \( Im(V_g) \) of group velocity represents the slow drift of the central wave number along the wave packet trajectory [40].

The reduced equations for the second harmonic components \( l = 2 \) at the second order \( m = 2 \), which arise from the nonlinear self-interaction of the carrier waves, are obtained in terms of \( (\varphi_l^{(1)})^2 \):
\[
n_2^{(2)} = D_1 (\varphi_2^{(1)})^2,
\]
\[
u_2^{(2)} = D_2 (\varphi_2^{(1)})^2,
\]
\[
\varphi_2^{(2)} = D_3 (\varphi_2^{(1)})^2, \tag{21}
\]
where \( D_1, D_2, \) and \( D_3 \) are presented in the appendix. Note that in the coefficient of \( \epsilon^2 \) \( (m = 2, l = 0) \), the relation \( \nu_l (\varphi_l^{(1)})^2 = 0 \) can be found from the ion-momentum equation. Because \( |\varphi_l^{(1)}| \) is of the order of \( \epsilon^2 \), \( \nu_l \) should be at least of the order of \( \epsilon \). Therefore, it will contribute to the coefficient of \( \epsilon^3 \) of the ion-momentum equation for \( m = 3, l = 1 \).

The nonlinear self-interaction of the carrier waves also results in the creation of the zeroth harmonic. Proceeding to the third order \( (m = 3, l = 0) \), we obtain
\[
n_3^{(2)} = D_4 (\varphi_3^{(1)})^2,
\]
\[
u_3^{(2)} = D_5 (\varphi_3^{(1)})^2,
\]
\[
\varphi_3^{(2)} = D_6 (\varphi_3^{(1)})^2, \tag{22}
\]
where \( D_4, D_5, \) and \( D_6 \) are given in the appendix.

Finally, the reduced equations for the third harmonic modes \( (m = 3, l = 1) \) yield an explicit compatibility condition in the form of the modified CGL equation:
\[
\frac{i}{\partial \tau} \Phi + \tilde{P} \frac{\partial^2 \Phi}{\partial \xi^2} + \tilde{Q} |\Phi|^2 \Phi + i \Upsilon \Phi = 0. \tag{23}
\]

Here, \( \Phi = \psi_{(1)} \), the dispersion coefficient \( \tilde{P} (= \text{Re}(\tilde{P}) + i \text{Im}(\tilde{P})) \), the nonlinear coefficient \( \tilde{Q} (= \text{Re}(\tilde{Q}) + i \text{Im}(\tilde{Q})) \), and the dissipative coefficient \( \Upsilon \) are:

\[
\tilde{P} = \frac{1}{2} \frac{\partial^2 \omega}{\partial k^2} = \frac{1}{\mathcal{A} k^2} (V^\omega D + i k^2 \sigma - k \sigma D)
- \nu \eta^* k^2 \epsilon + \omega \eta^* k^2 - k G \epsilon - i \omega^2 - \omega \epsilon + k \eta^* A \omega + i V \eta^* k^2 \nu D),
\tilde{Q} = \frac{2 \nu (i k^2 \sigma + \omega \eta^* k^2 - i \omega^2)}{\mathcal{A} k (2 i \omega - \eta^* k^2)} (\Delta_3 + \Delta_0) + \omega (\Delta_1 + \Delta_4),
\Upsilon = \frac{\nu \omega}{\eta^* k^2 + 2 \omega},
\]

where \( \mathcal{A}, D, \mathcal{E}, \mathcal{F}, \mathcal{G}, \text{ and } \Delta_1-\Delta_4 \) are given in the appendix. Note that the presence of linear dissipative term \( i \Upsilon \Phi \) in the modified CGL equation (23) is because of the collision between ions and neutral particles.

It is to be noted that the nonlinear evolution equation in the weakly coupled plasma is the Korteweg-de Vries (KdV) equation rather than the CGL equation in this paper. This difference can be explained as follows. In weakly coupled plasma, the individual particles interact almost exclusively through the macroscopic mean-fields [43] which means that the viscous dissipation effect and memory effect vanish. In this limit, the generalized hydrodynamic model reduces to the conventional fluid equations. In the long-wavelength regime, the evolution equation governing the longitudinal ion-acoustic waves then takes the form of the KdV equation [44] which can be derived by the reductive perturbation method. In our present work, we investigate the modulation instability of ion-acoustic waves in the strong-coupling limit in the hydrodynamic regime. In contrast to the weakly coupled plasma, the strongly coupled plasma suffers viscous dissipation. Furthermore, in the short-wavelength regime, the slow parallel modulation of finite amplitude monochromatic plane waves can lead to the formation of an envelope pulse. Therefore, the evolution equation in our work takes the form of the CGL equation rather than the KdV equation. In other words, different characteristic evolution equations for small but finite amplitude ion-acoustic waves can be obtained in various propagation regimes.

IV. Modulation instability and rogue waves

In this section, we will investigate the modulation instability of ion-acoustic waves in our plasma model. In addition, two different kinds of second-order ion-acoustic rogue waves are discussed within the unstable region.

IVA. Modulation instability

Consider the development of small modulation \( \delta \psi = \delta \psi(\xi, \tau) \) according to

\[
\Phi = (\Phi_0 + \delta \psi) \exp \left(-i \int_0^\tau \Delta(\tau) d\tau \right), \tag{24}
\]

where \( \Phi_0 \) is the constant amplitude of pump carrier waves, \(|\Phi_0| \gg |\delta \psi|, \Delta(\tau) \) is the nonlinear frequency shift produced by nonlinear reaction. Substituting equation (24) into the modified CGL equation (23), we obtain the following relations by collecting the terms in the zeroth and first order:

\[
\Delta(\tau) = -\tilde{Q} |\Phi_0|^2, \tag{25}
\]

\[
i \frac{\partial \delta \psi}{\partial \tau} + \tilde{P} \frac{\partial^2 \delta \psi}{\partial \xi^2} + \tilde{Q} |\Phi_0|^2 (\delta \psi + \delta \psi^*) + i \Upsilon \delta \psi = 0, \tag{26}
\]

where \( \delta \psi^* \) is the complex conjugate of \( \delta \psi \). Substituting the following equation

\[
\delta \psi(\tau, \xi) = U e^{i K^\xi - \Omega^\tau} + V e^{-i K^\xi - \Omega^\tau} \tag{27}
\]

into equation (26), we obtain the Lange–Newell’s criterion [41] for MI: The plane waves are supercritical (i.e. unstable) for \( \text{Re}(\tilde{P}) \text{Re}(\tilde{Q}) + \text{Im}(\tilde{P}) \text{Im}(\tilde{Q}) > 0 \), and are subcritical (i.e. stable) for \( \text{Re}(\tilde{P}) \text{Re}(\tilde{Q}) + \text{Im}(\tilde{P}) \text{Im}(\tilde{Q}) < 0 \). In equation (27), \( U \) and \( V \) are complex constant amplitudes, \( \Omega^* \) denotes the complex conjugate of \( \Omega, K|K| < k \) and \( \Omega|\Omega| < \omega \) represent the wave number and angular frequency of the modulation, respectively.

To study the roles of the dissipative effect, i.e. the ion kinematic viscosity on ion-acoustic waves, we plot the variations of \( \text{Re}(\tilde{P}) \text{Re}(\tilde{Q}) + \text{Im}(\tilde{P}) \text{Im}(\tilde{Q}) \) for different values of \( \eta^* \) in figure 1. Obviously, increasing the values of \( \eta^* \) can make the unstable region narrower (i.e. \( \text{Re}(\tilde{P}) \text{Re}(\tilde{Q}) + \text{Im}(\tilde{P}) \text{Im}(\tilde{Q}) > 0 \)), while the stable region

**Figure 1.** The effects of \( \eta^* \) on \( \text{Re}(\tilde{P}) \text{Re}(\tilde{Q}) + \text{Im}(\tilde{P}) \text{Im}(\tilde{Q}) \). The parameters are \( q = 1.8, \sigma = 0.31, a = 0.01 \).
(i.e. \( \text{Re}(\hat{P})\text{Re}(\hat{Q}) + \text{Im}(\hat{P})\text{Im}(\hat{Q}) < 0 \)) becomes wider. In other words, increasing the strength of the dissipative effect leads to the shrinkage (expansion) of the unstable (stable) region.

When we neglect the ion-neutral particle collision and the viscous dissipation effect (for example, when the viscosity contribution is quite small compared to the compressibility viscous dissipation effect (for example, when the viscosity), the modified CGL can be reduced to the following standard NLS equation

\[
i \frac{\partial \Phi}{\partial t} + P \frac{\partial^2 \Phi}{\partial x^2} + Q|\Phi|^2 \Phi = 0, \tag{28}\]

where \( P = \text{Re}(\hat{P}) \) and \( Q = \text{Re}(\hat{Q}) \). According to Lange–Newell’s criterion, we can immediately see that the amplitude-modulated envelope will be stable for \( PQ < 0 \) in the presence of a small perturbation. For this case, the ion-acoustic waves may propagate in the form of dark-type (‘black’ or ‘gray’) excitations with vanishing or finite amplitudes in the center. On the other hand, when \( PQ > 0 \) the carrier wave is unstable particularly to external perturbations with the threshold wave number \( k_c^2 = 2Q|\Phi_0|^2/P \). For this unstable case, the carrier wave may either collapse or blow up because of the external perturbations, or lead to the localized pulse-shaped envelopes confining the fast carrier wave.

Now, we study the instability growth rate of ion-acoustic waves for \( PQ > 0 \). Substituting \( \Omega = \Omega^\gamma \) into the following nonlinear dispersion relation

\[
\Omega^2 = (PK^2)^2 \left[ 1 - \frac{2Q|\Phi_0|^2}{PK^2} \right],
\]

we obtain the instability growth rate

\[
\Gamma = PK^2 \sqrt{\frac{2Q|\Phi_0|^2}{PK^2} - 1}. \tag{29}\]

Taking the derivative of equation (29) with respect to \( K \) and setting it to zero, we obtain \( K_{\text{max}} = \sqrt{Q|\Phi_0|^2}/P \). With this value of \( K_{\text{max}} \), we obtain the maximum growth rate of the instability \( \Gamma_{\text{max}} = |Q| |\Phi_0|^2 \).

To investigate how the plasma parameters including nonextensive index \( q \), nonthermal index \( a \), and effective temperature \( \sigma \) affect the modulational unstable \((PQ > 0)\) and stable \((PQ < 0)\) regions, we depict the curves representing the loci of all points for \( PQ = 0 \) in figure 2:

(i) Plot (a) shows that an increase of \( q \) can lead to an increase of the critical wave number \( k_c \), which makes the unstable (stable) region smaller (bigger).

(ii) Plot (b) shows that the critical wave number \( k_c \) decreases slightly with an increase of \( a \), which means that the stable region becomes slightly smaller.

(iii) With increasing values of \( \sigma \), the unstable region expands and the critical wave number \( k_c \) decreases, which is presented in plot (c).

IV.B. Rogue waves

Now, we will analyze an interesting modulational unstable solution [42] of NLS equation (28) for \( PQ > 0 \),

\[
\Phi(\xi, \tau) = \sqrt{\frac{P}{Q}} \left[ 1 + \frac{R_2(\xi, \tau) + iS_2(\xi, \tau)}{T_2(\xi, \tau)} \right] e^{iP\tau}, \tag{30}\]

where

\[
R_2(\xi, \tau) = 36 - 48\xi^4 - 144\xi^2(4(\tau^2))^2 + 1 - 24\sqrt{2} \mu_1 \xi - 960(\tau r)^2 - 864(\tau r)^2 + 48\mu_2 \tau r, \]

\[
S_2(\xi, \tau) = 24 \left[ (15 - 4\xi^4 + 12\xi^2 - 2\sqrt{2} \mu_1 \xi) - 16(\tau r)^2 \right] - 8(\tau r)^2(2\xi^2 + 1) + \mu_1 \left[ 2(\tau r)^2 - \xi^2 - \frac{1}{2} \right], \]

\[
T_2(\xi, \tau) = 8\xi^6 + 12\xi^4(4(\tau r)^2 + 1) + 6\xi^2(3 - 4(\tau r)^2)^2 + \mu_1 \left[ \mu_1 + 2\sqrt{2} \xi(12(\tau r)^2 - 2\xi^2 + 3) \right] + \mu_2 \left[ \mu_2 + 4(6\xi^2 - 4(\tau r)^2) + 432(\tau r)^4 \right] + 396(\tau r)^2 + 9 + 64(\tau r)^6. \]
In [12], the ion-acoustic waves were experimentally observed in the strongly coupled regime by photoionization of laser-cooled strontium atoms from a magneto-optical trap. Therefore, we choose the plasma parameters corresponding to the above plasma experiment for our graphical analysis of rogue waves, i.e. the densities $n_{e0} \approx n_{i0} = 10^{15}$ cm$^{-3}$, ion charge state $Z_i = 1$, electron temperature $T_e = (1-1000)$ K, ion temperature $T_i \approx 1$ K, and the Debye length $\lambda_D \sim (3-30)$ $\mu$m.

Because we discuss the ion-acoustic waves in the hydrodynamic regime, the effective ion temperature $T_{\text{eff}}$ dominates over the ion temperature $T_i$ with the parameters satisfying $1 < \kappa < 7$ and $170 \lesssim \Gamma_i \lesssim 200$.

Note that the above second-order ion-acoustic rogue wave solution (30) contains two free parameters $\mu_1$ and $\mu_2$ which play important roles in the formation of the waves:

**Type I:** When $\mu_1 = \mu_2 = 0$, the second-order ion-acoustic rogue wave has one single peak. The position of the peak is $(0, 0)$ on the $(\xi, \tau)$-plane. From figure 3, we can see this type of ion-acoustic rogue wave is localized both in time and space.

**Type II:** When $\mu_1 \neq 0$ or $\mu_2 \neq 0$, the second-order ion-acoustic rogue wave splits into three Peregrine breathers (the first-order rogue waves). The free parameters $\mu_1$ and $\mu_2$ are used to describe the relative positions of these three first-order breathers in the rogue wave triplet. For simplicity, we set $\mu_1 = 0$ and $\mu_2 = 0$ in this paper. On the $(\xi, \tau)$-plane, the positions of the three wave peaks which are denoted by $A$, $B$, and $C$ are given as:

$$A : \left( \frac{\sqrt{2}}{2} \mu_1^{1/3}, 0 \right), \quad B : \left( -\frac{\sqrt{2}}{2} \mu_1^{1/3}, \frac{\sqrt{2}}{2} \mu_1^{1/3} \right), \quad C : \left( -\frac{\sqrt{2}}{2} \mu_1^{1/3}, -\frac{\sqrt{2}}{2} \mu_1^{1/3} \right).$$

Here, the radial distance of each peak from the origin $(0, 0)$ is $r = \frac{\sqrt{2}}{2} \mu_1^{1/3}$. For a better understanding, we depict this type of ion-acoustic rogue wave in figure 4. Obviously, this kind of rogue wave contains three shifted first-order doubly-localized solutions where each is located on a circle with 120 degrees angular separation among these three peaks. Moreover, peak $B$ is symmetric with peak $C$ which indicates that they have the same structure.
In figure 5, we present how the parameters $q$, $a$, and $\sigma$ influence the nonlinear structure of ion-acoustic rogue wave triplets (type II). Clearly, we can see that the amplitudes of the peaks $B$ and $C$ are affected by these plasma parameters:

(i) Plot (a) indicates that an increase of $q$ value leads to an enhancement of the rogue wave amplitude. It means that increasing the strength of nonextensivity of electron velocity distribution can enhance the nonlinearity of the plasma system and concentrate a significant amount of energy, which makes the pulses taller.

(ii) Plots (b) and (c) show that the effects of $a$ and $\sigma$ on the structures of rogue wave triplets are opposite to that of $q$. In others words, the amplitudes of peaks $B$ and $C$ shrink when the values of $a$ and $\sigma$ increase. We speculate that these interesting phenomena can be explained as follows: the increases of nonthermal electrons number and effective ion temperature can shrink the nonlinearity of the system, and therefore the pulses become shorter.

These plasma parameters have the same qualitative effect on the ion-acoustic rogue wave with single peak (type I) as that on the rogue wave triplet (type II). For simplicity, we do not present the figures here.

V. Conclusions

In summary, the nonlinear propagation of small amplitude ion-acoustic waves is investigated in a collisional plasma composed of strongly coupled ions and nonthermal nonextensive electrons. The strongly coupled plasma is modeled by the generalized hydrodynamic equations in the integro-differential form, which can be simplified to a set of nonlinear basic equations by employing the appropriate physical ansatz.

We investigate the modulation instability of ion-acoustic waves propagating in the strongly coupled plasma medium. The dynamics of modulated ion-acoustic waves is governed by the modified complex GLE with a linear dissipative term. The unstable (stable) region becomes narrower (wider) when the values of $\eta^*$ (ion kinematic viscosity) increase, which means that the plasma system dissipates the wave energy and hence a stable envelope wave packet will be the dominant feature of the amplitude-modulated ion-acoustic pulse.

If we neglect the dissipative effect, the modified CGL equation reduces to the NLS equation with real coefficients. The effect of the parameters containing nonextensive index $q$, nonthermal index $a$, and effective temperature ratio $\sigma$ on the stable/unstable regions is discussed in detail. Within the unstable region, the rogue wave can be created because of the modulation instability. Our results reveal that the stronger nonextensivity of electrons can increase the nonlinearity of the system and concentrate a significant amount of energy, which makes the amplitude of the rogue wave triplet taller. However, increasing the values of $a$ and $\sigma$ leads to shrink the nonlinearity of the system and disperse its energy, and therefore the pulses of ion-acoustic rogue waves become shorter.

We hope that the present work can provide a good fit between theoretical analysis and real applications in laboratory plasma experiments.

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Appendix. Expressions of the coefficients

\[ A = \frac{-ik}{(\omega k^2 - i\omega k + \omega k^2)}, \quad B = (\sigma A k - V_g A k + 1 - 2i\eta^* A k), \quad C = (\eta^* k^2 - \omega), \]

\[ D = \frac{-iA(\omega) + iA(C)k}{(\omega + i\omega)}, \quad E = -\frac{(\omega B + A \omega k - A V_g^2 k^2)}{(\omega + i\omega^2)}, \quad F = (4\eta^* k^2 - 2i\omega), \quad \mathcal{G} = (-V_g - 2i\eta^* k), \]

\[ \Delta_1 = \frac{k_c f_{\omega} A k + 4k_c f_{\omega} A k^2}{k_c f_{\omega} A k + 2k_c f_{\omega} A k^2} = \frac{1}{2k_c f_{\omega} A k + 2k_c f_{\omega} A k^2}, \]

\[ \Delta_2 = -\omega - \frac{2ik_c f_{\omega} A k + 2i(k_c f_{\omega} A k^2 + 2i(k_c f_{\omega} A k^2 + 8i(k_c f_{\omega} A k^2 + 1 + iA c k^2))}{(2i k_c f_{\omega} A k + 2i(k_c f_{\omega} A k^2 + 8i(k_c f_{\omega} A k^2 + 1 + iA c k^2))}, \]

\[ \Delta_3 = \frac{c_1 A_c^2 - 2c_1 A_c k^2}{c_1 A_c^2 + 2c_1 A_c k^2}, \quad \Delta_4 = \frac{c_1 A_c^2 - 2c_1 A_c k^2}{c_1 A_c^2 + 2c_1 A_c k^2}, \]

\[ \Delta_5 = \frac{V_c c_1 A_c^2 - 2 V_c c_1 A_c k^2 + 2 A_c^2 k^2}{V_c c_1 A_c^2 + 2 V_c c_1 A_c k^2 + 2 A_c^2 k^2}, \quad \Delta_6 = \frac{V_c c_1 A_c^2 - 2 V_c c_1 A_c k^2 + 2 A_c^2 k^2}{V_c c_1 A_c^2 + 2 V_c c_1 A_c k^2 + 2 A_c^2 k^2}. \]

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