Multiple-harmonic amplitude and phase control method for active noise and vibration reshaping

Jinxin Liu¹,2, Liangdong Yang¹,2, Laihao Yang¹,2, Xingwu Zhang¹,2 and Xuefeng Chen¹,2

Abstract
With the development of active noise and vibration control technology, there are increasing demands for active noise and vibration reshaping (ANVR) other than cancellation in the fields of active sound quality control, psychoacoustics, medical devices, military equipment, etc. The active noise equalizer (ANE) is one of the most popular ANVR methods. However, the ANE only controls (scales) the amplitude of the residual noise without considering the control of phase, and is unable to control frequency components that are excluded in primary noise. In this paper, we propose a more general and effective algorithm, called amplitude and phase control (APC), for ANVR, which can control amplitude and phase, simultaneously, of any interesting frequency component of the residual noise. Firstly, a modified active noise equalizer (MANE) algorithm with improved convergence is proposed based on the reference amplitude normalization and structure modification. Then, the APC algorithm is derived based on the MANE algorithm. Furthermore, numerical simulations and analytical investigations of the APC algorithm are conducted. Finally, case studies using experimental data are carried out. The multiple-harmonic primary noise was measured from a rotor test-platform and the secondary path model was identified according to a random vibration test on a shell structure. The results further verify the validity and robustness of the proposed APC algorithm.

Keywords
Active noise and vibration control, active noise and vibration reshaping, spectral reshaping, amplitude and phase control, noise and vibration injection

1. Introduction
Active noise and vibration control (ANVC) is now a very popular and promising technology in the field of noise and vibration control. It generates a secondary noise/vibration to counteract the unwanted primary noise/vibration by the superposition principal. In the present literature and application reports, active noise/vibration control has many aliases, such as active noise/vibration cancelation, suppression, reduction, damping, rejection, isolation, elimination and attenuation, all of which are aimed at reducing or eliminating the unwelcome noise and vibration (Alkhateb and Golnaraghi, 2003; Iorga et al., 2008; Casciati et al., 2012; Kajikawa et al., 2012; Akraminia and Mahjoob, 2016; Aslan and Paurobally, 2016; Liu et al., 2016a, 2016b).

However, with the development of active control technology, some residual noise/vibration spectral reshaping requirements have arisen. In psychoacoustics, it is believed that not all the noise is unwelcome and people prefer certain types of sound or residual noise (Fastl and Zwicker, 2007).

Taking the internal noise of an automobile as an example, some of the internal noises will cause unpleasant feelings to the passengers and drivers, while some other noises that reveal the running condition of the machine are desired by drivers or technicians. Usually, the sound related to engine speed is the most
desired internal noise since it provides acoustic feedback with regard to vehicle speed and acceleration to the driver (Kuo et al., 2007; De Oliveira et al., 2010). From the viewpoint of the automobile manufacturer, they consider certain internal noise as brand individuality, leading to different types of products, such as sportiveness, refinement and luxury (Cerrato, 2007, 2009).

Therefore, good internal noise is more than being quiet. People hope the noise can be partly reduced or even be artificially designed, which motivates the development of sound and vibration quality control (SVQC) in the Noise, Vibration and Harshness (NVH) engineering of an automobile. Another non-automobile example is wearable medical device. On one hand, people want some reassuring feedbacks (tactile or auditory) which indicate that “everything is working as it should”, but on the other hand it cannot interfere with sleep, comfort or privacy (Pietila and Cerrato, 2012). More particularly, for some military equipment, it is expected that the radiated vibration or noise could be artificially defined or modified for the purpose of anti-reconnaissance (Liu and Chen, 2016).

The first idea of spectral reshaping of residual noise, proposed by Kuo in the 1990s (Kuo and Ji, 1994; Kuo and Tsai, 1994), was called the adaptive noise equalizer (ANE). Numerous follow-up studies, such as the frequency domain ANE (Kuo and Tahernezhadi, 1997), the multi-channel ANE (De Diego et al., 2004; Gonzalez et al., 2006), the normalized ANE (NANE; Kuo et al., 2007; De Oliveira et al., 2010) and the enhanced ANE (Liu and Chen, 2016), have been conducted. The ANE is substantially an extension of the famous narrowband FXLMS algorithm for active noise control (Kuo and Morgan, 1996). The basic idea of the ANE is to feed a well-designed pseudo-error signal (rather than a real-error signal) back to the adaptive controller. Thus, when the performance function that is related to the pseudo-error is optimized, the real-error (residual error) will reach a pre-defined target. Kuo first used the name “residual noise shaping” (Kuo and Tsai, 1994). Other names, such as active sound-profiling (Rees and Elliott, 2006) and active noise reshaping (De Diego et al., 2004), have also been used in the follow-up studies. To emphasize applications in the fields of both noise and vibration, we use the name active noise and vibration reshaping (ANVR) throughout this paper.

ANE-type algorithms suffer from two drawbacks. One is that they only aim at controlling (scaling) the amplitude of residual noise without considering the control of the phase. However, in real applications, phase control is required. For example, in psychoacoustics and SVQC, except for loudness (the amplitude), auditory roughness is also a very important descriptor. There are studies that show that the phase difference can become audible and thus clearly affect the roughness perception, even when keeping the amplitude spectrum constant (Pressnitzer and McAdams, 1999; Fastl and Zwicker, 2007; Mosquera-Sanchez and De Oliveira, 2014). The other drawback is that they are unable to control the frequency components that are not included in the primary noise (i.e. inject new frequency components). However, sometimes noise and vibration injection (NVI) is required. For example, in the electrical vehicle industry, the lack of engine noise makes the sounds of other subsystems (pumps, compressors, fans, etc.) very noticeable. Also, the lack of acceleration acoustic feedback makes the vehicles feel not “cool”. So, it is desirable to reduce the noise of subsystems as well as make some other “cool” engine sounds. In fact, one strategy adopted by manufacturers is to inject pleasant powertrain sounds (Cerrato, 2009; Samarasinghe et al., 2016).

In this paper, we propose a more general ANVR algorithm, called the amplitude and phase control (APC) algorithm, which can simultaneously control the amplitude and phase of any interesting frequency component (including the application of NVI). Firstly, we give modifications to the traditional ANE for a larger stability region and uniform convergence rate in multiple-harmonic applications. Then, the APC is derived according to the modified ANE. Numerical simulations and analytical investigations are conducted to verify the superiority and validity of the APC algorithm. Case studies using experimental data are conducted to verify the robustness of the APC algorithm. This paper is structured as follows: Section 2 derives a modified active noise equalizer (MANE) algorithm considering the improvement of convergence; Section 3 derives the APC algorithm and conducts numerical and analytical analysis. Section 4 carries out case studies using the experimental data. Section 5 briefly concludes this paper.

2. A modified active noise equalizer

2.1. Internal-model-based ANVR method

The ANE, proposed by Kuo (Kuo and Ji, 1994), is one of the most popular methods for ANVR. The structure of the ANE is shown in Figure 1(a). The basic structure of the ANE is similar to that of the famous FXLMS algorithm (Kuo and Morgan, 1996). The differences are that (1) the ANE feeds a pseudo-error signal $e_n(n)$, rather than a real-error signal $e(n)$, back to the least mean squares (LMS) algorithm; (2) the ANE contains two branches, that is, a canceling branch and
a balancing branch, into whom the gains 1−β and β are inserted. From the block diagram in Figure 1(a), the real-error and pseudo-error signal of the ANE system are

\[ e(n) = d(n) - (1-\beta) y(n) * s(n) \]  

(1)

and

\[ e_s(n) = e(n) - \beta y(n) * \hat{s}(n) \]  

(2)

respectively, where s(n) represents the impulse response of the secondary path, \( \hat{s}(n) \) denotes the identification of s(n) and * indicates convolution. With the convergence of the LMS algorithm, the pseudo-error \( e_s(n) \) approaches zero. Consider that s(n) is equal to \( \hat{s}(n) \); from equations (1) and (2), there is \( s(n)*y(n) = d(n) \). Thus, the steady-state real-error is

\[ e(n) = \beta d(n) \]  

(3)

It can be seen from equation (3) that the ANE actually scales the primary noise \( d(n) \) by the gain factor β. By specifying the gain factor, the ANE can work in the cancellation mode (\( \beta = 0 \)), suppression mode (0 < \( \beta < 1 \)), neutral mode (\( \beta = 1 \)) or enhancement mode (\( \beta > 1 \)).

Rees and Elliot (2006) suggested that the ANE is actually equivalent to the internal model FXLMS (seen in Figure 1(b)). The internal model FXLMS estimates the primary noise, and then uses the filtering or scaling of the estimated primary noise to design pseudo-error signal for the LMS algorithm. From the block diagram in Figure 1(b), the estimated primary noise is

\[ \hat{d}(n) = e(n) + \hat{s}(n) * y(n) = d(n) - [s(n) - \hat{s}(n)] * y(n) \]  

(4)

Figure 1. Internal-model-based active noise and vibration reshaping: (a) classical structure of the active noise equalizer (ANE); (b) alternative block diagram of the ANE.

If s(n) = \( \hat{s}(n) \), then \( \hat{d}(n) = d(n) \). Thus, the pseudo-error of the system is

\[ e_s(n) = e(n) - \beta \hat{d}(n) = e(n) - \beta d(n) \]  

(5)

With the convergence of the LMS algorithm, \( e_s(n) \rightarrow 0 \), so the steady-state real-error \( e(n) \) is equal to \( \beta d(n) \). Comparing with equation (3), it can be seen that the ANE is indeed equivalent to the internal model FXLMS algorithm, so we name the ANE and its variants as the internal-model-based ANVR method in this paper. The internal-model-based ANVR method will inherit the advantages and disadvantages of the traditional FXLMS algorithm. Therefore, the first obstacle should be the relatively slow convergence resulting from the secondary path (Elliott, 2000).

### 2.2. MANE for convergence improvement

There are several convergence improvement methods for the FXLMS algorithm. Some of them aim at conditioning the reference signal by filtering, scaling or employing different filter structures (such as, the lattice filter, sub-band filter and orthogonal transform) (Kuo and Morgan, 1996; Kuo et al., 2007; Chen et al., 2015). Some others use alternative adaptive mechanisms, such as the variable-step-size algorithm, Newton algorithm, Kalman algorithm and recursive least squares (RLS) algorithm (Kuo and Morgan, 1996; Wang et al., 2014). For this study, there are two methods that draw our attentions. They are the modified FXLMS (Bjarnason, 1992; Elliott, 2000) and the NANE (Kuo et al., 2007), both of which are variants of the traditional FXLMS algorithm. Figure 2 shows diagrams of the traditional, rearranged and modified FXLMS, where the rearranged FXLMS is an equivalent system of the traditional FXLMS under the assumption of linear time-invariant.

Note that the LMS-type algorithms are actually time-variant, since the controller keeps changing...
is the error signal, and

$$e(n) = d(n) - s(n) + \sum_{j=1}^{2q} w_j(n) x_j(n)$$

(7)

is the error signal, and

$$x_j(n) = \begin{cases} A_j \cos(\omega_j n), & j = 2i - 1, \\ A_j \sin(\omega_j n), & j = 2i, \end{cases} \quad (i = 1, 2, \ldots, q)$$

(8)

is the jth sinusoidal reference signal. In equation (8), $A_j$ is the amplitude of the jth reference signal, $\omega_j$ is the jth harmonic of the primary noise. Consider that the real secondary path is precisely equal to its model, that is, $s(n) = \hat{s}(n)$, the rearranged FXLMS in Figure 2(b) can be obtained by exchanging the position of the secondary path and the controller. Then, the error signal can also be expressed as

$$e(n) = d(n) - \sum_{j=1}^{2q} w_j(n) x_j(n) \ast \hat{s}(n)$$

(9)

Substituting equation (9) into equation (6), there is

$$w_j(n + 1) = w_j(n) - \mu_j x_j(n) \ast \hat{s}(n)$$

(10)

where

$$\hat{s}(n) = x_j(n) \ast \hat{s}(n)$$

(11)

is the filtered reference signal. Taking the expectation of both sides of equation (10), there is

$$E[w_j(n + 1)] = E[w_j(n)] - \mu_j E[x_j(n)] E[\hat{s}(n)]$$

$$+ \mu_j E[d(n) \hat{s}(n)]$$

$$= k_j E[w_j(n)] + \mu_j E[d(n) \hat{s}(n)]$$

(12)

where

$$k_j = 1 - \mu_j \frac{2}{E[\hat{s}^2(n)$$]

(13)

Equation (12) will converge if $|k_j| < 1$, that is,

$$0 < \mu_j < \frac{2}{E[\hat{s}^2(n)$$]

(14)

In real applications, since the instantaneous-square (instead of mean-square) error is applied for coefficient updating, a more practical step-size bound is

$$0 < \mu_j < \mu_{\text{max}} = \frac{2}{\max \left[ \frac{\hat{s}^2(n)}{2} \right]} = \frac{2}{A_j A_j^2}$$

(15)

where $A_j s$ is a scaling ratio of the jth reference signal introduced by the secondary path model, $\hat{s}(n)$. In the traditional FXLMS, the amplitudes of sinusoidal

\[\text{Figure 2. Traditional, rearranged and modified FXLMS: (a) traditional; (b) rearranged; (c) modified.}\]
reference signals usually have a unit value, that is, $A_j = 1$, so the maximum step-size can be further expressed as

$$\mu_{\text{max}} = \frac{2}{A_j^2}$$

(16)

The maximum step-size in equation (16) is derived based on the rearranged FXLMS. Unfortunately, it does not apply to the traditional FXLMS, since the slow adaptation assumption is no longer true when the step-size is large. The secondary path between the controller and the error signal will introduce delay to the system, which strongly affects the step-size bound of the traditional FXLMS. Elliott and Nelson (Elliott, 2000) suggested that the upper limit of the step-size of the traditional FXLMS is approximately determined by

$$\mu_{\text{max}} \approx \frac{2}{(I + \Delta)^{0.2}}$$

(17)

rather than

$$\mu_{\text{max}} \approx \frac{2}{I^{0.2}}$$

(18)

where $\Delta$ is the delay of the secondary path, $I$ is the number of controller coefficients and $\bar{r}^2$ is the mean-square value of the filtered reference signal. For the parallel structure of multiple-frequency FXLMS, each coefficient is tuned separately, so $I$ is equal to 1. Equations (17) and (18) are estimated by simulations with white noise reference signals (Elliott, 2000: 137). However, for the sinusoidal reference signal, it is more realistic to consider $\bar{r}^2$ as the maximum-square (rather than mean-square) value of the filtered reference signal. Therefore, under the particular case of the sinusoidal reference signal, equation (18) is equivalent to equation (15). Comparing equation (17) with equation (15), the traditional FXLMS has a more stringent step-size bound than the rearranged version due to the delay in the secondary path.

Although the rearranged FXLMS has a larger convergence region, it is invalid in real applications since the “real” secondary path cannot be exchanged with the controller. The modified FXLMS in Figure 2(c) is actually a realization of the rearranged FXLMS. In the modified system, the estimated primary noise is

$$\hat{d}(n) = d(n) - y(n) * [s(n) - \hat{s}(n)] = d(n)$$

(19)

which is not affected by the adaptation process. Thus, the estimated error signal is equal to that in equation (9), that is

$$\hat{e}(n) = d(n) - \sum_{j=1}^{2g} w_j(n) [x_j(n) * \hat{s}(n)]$$

(20)

without the assumption of slow adaptation. Therefore, the step-size bound in equation (15) will apply to the modified FXLMS. It can be seen from Figure 2(c) that the plant delay between the adaptive controller and the observation of the error has been removed by the modified structure.

Simulation on a pure delayed secondary path

$S(z) = z^{-\Delta}$

(21)

is conducted to verify the step-size bound in equation (15), where $\Delta$ is the sample number of delay. The simulation parameters are listed in Table 1. The results are shown in Figure 3. It can be seen that (1) even one step delay will dramatically narrow the step-size bound of the traditional FXLMS; (2) the upper bound in equation (15) applies to the rearranged and the modified FXLMS (e.g. they diverge under 100% of $\mu_{\text{max}}$ and converge under 95% of $\mu_{\text{max}}$); (3) traditional, rearranged and modified FXLMS are equivalent under slow adaptation (e.g. under 10% of $\mu_{\text{max}}$).

For multiple-harmonic control, the convergence property is also influenced by the amplitude-frequency characteristic of the secondary path model. In the FXLMS algorithm, the reference signals are filtered by the secondary path model, leading to an amplitude scaling and a phase shift. The phase shift is useful for the LMS algorithm, but the amplitude scaling is not necessary. Since the amplitude tuning of different frequency components is different, the step-size is restricted by the smallest amplitude response of the

### Table 1. Simulation parameters for traditional, rearranged and modified FXLMS.

<table>
<thead>
<tr>
<th>Items</th>
<th>Traditional</th>
<th>Rearranged</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sampling freq. (Hz)</strong></td>
<td>1024</td>
<td>1</td>
<td>1024</td>
</tr>
<tr>
<td>**Sampling Time (s)</td>
<td>1</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>**Noise d(n)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Freq. (Hz)</strong></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Mag. (dB)</strong></td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td><strong>Phases (deg.)</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>S(2)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Step-size</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$S(z) = z^{-\Delta}$

(21)
secondary path model among all frequencies that are considered. A good solution is to give an amplitude rectification (normalization) to the reference signal. Let

$$A_j = \frac{p_m}{A_{j_k}}, \quad p_m = \max_{j=1,2,...,2q} |A_{j_k}|$$

(22)

where $p_m$ is the largest amplitude response of secondary path among all the frequencies that are considered. Substituting equation (22) into equation (15), the step-size bound becomes

$$0 < \mu_j < \frac{2}{(A_j p_m / A_{j_k})^2} = \frac{2}{p_m^2}$$

(23)

It can be seen that the upper bounds of different frequency components are the same after rectification. Thus, a uniform step-size can be applied for all components (i.e. replace $\mu_j$ in equation (6) with $\mu$).

The above modifications can be applied to the ANE system. The reference rectification has been applied to the ANE by Kuo et al.(2007), which is referred as the NANE. We further apply the modified structure to the NANE according to the alternative structure of the ANE in Figure 1(b), since both of them have an estimation of the primary noise. Therefore, a MANE that considers the convergence improvement is proposed, as shown in Figure 4. From the diagram in Figure 4, a pseudo-error signal

$$e(n) = \hat{e}(n) - \beta \hat{d}(n) = e(n) - \beta d(n)$$

(24)

is fed back to the system instead of real one. With the convergence of the LMS algorithm, the pseudo-error signal approaches zero. According to equation (24), there is

$$e(n) = \beta d(n)$$

(25)
Table 2. Simulation parameters for comparison of the active noise equalizer (ANE), normalized active noise equalizer (NANE) and modified active noise equalizer (MANE).

<table>
<thead>
<tr>
<th>Items</th>
<th>Sampling Freq.(Hz)</th>
<th>Sampling Time (s)</th>
<th>Noise d(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANE,NANE and MANE</td>
<td>1024</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Freq. (Hz)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mag.(dB)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Phase (deg.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Gain</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Step-size</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1</td>
<td>90</td>
</tr>
</tbody>
</table>

It can be seen that the steady-state response of the MANE is identical to that of the ANE. Substituting equation (24) into equation (10), the updating equation of the MANE is

\[
w_j(n+1) = w_j(n) - \mu_j \hat{x}_j(n) \sum_{j=1}^{2p} w_j(n) z_j^2(n) + \mu_j \beta d(n) \hat{x}_j(n)\]

(26)

Comparing equation (26) with equation (10), the difference is that there is a scaling factor \(\beta\) in the last right-hand term. However, the convergence property is not affected, since the first two right-hand terms are the same. Therefore, the step-size bound of the MANE obeys equation (16). Furthermore, it will obey equation (23) if reference rectification is applied.

The step-size bound of the traditional ANE is the same as that of the traditional FXLMS (Kuo and Morgan, 1996). The NANE improved the transient property by a uniform the convergence rate among the different frequency components. The MANE further improved the transient property by synthesizing both the modified FXLMS and the NANE. A numerical investigation of the transient properties of the ANE, NANE and MANE is conducted on a second-order secondary model

\[
S(z) = 0.01 \times \frac{z^{-1} + 0.99z^{-2}}{1 - 1.95z^{-2} + 0.98z^{-2}}
\]

(27)

The simulation parameters are listed in Table 2. The primary noise consists of two sinusoidal signals with predefined amplitude and phase. The gain factor \(\beta = 0.5\) indicates suppression of the primary noise by 50%.

Three different step-sizes, that is, 0.001, 0.002 and 0.005, are considered in this simulation. Figure 5 shows the real-error signal and pseudo-error signal of different algorithms. For display convenience the curves of the NANE and MANE are cut in half. It can be seen that in the case of slow tuning (when step-sizes are 0.001 and 0.002), the convergence of the NANE and MANE are similar and superior to that of the ANE.

However, the stability regions of the NANE and ANE are restricted by the secondary path. As a result, in the case of fast tuning (when the step-size is 0.005), the NANE and ANE are divergent, while the MANE is still convergent with an even faster rate. Although the steady-state response of the MANE is derived under the assumption of slow tuning, the simulation results show that the transient (convergence) property of the MANE is indeed improved by normalizing the different frequency components and extending the stability region.

3. Amplitude and phase control method

3.1. Algorithm formulation

The ANE-type (e.g. ANE, NANE and MANE) methods actually use the scaling of primary noise as the target signal to design the pseudo-error signal, which keeps the phase the same as the primary noise. As a consequence, they have no phase control ability. Some further modifications can be made to extend the phase control ability. A direct idea is to use a filter rather than a gain factor to condition the estimated primary noise. Suppose the feedback filter is \(c(n)\), then the target signal will be \(c(n) \ast d(n)\).

From the derivations in Section 2, the real-error signal becomes

\[
e(n) = c(n) \ast d(n)
\]

(28)

If the frequency response of \(c(n)\) is well-designed, the amplitude and phase of \(d(n)\) can be controlled. However, such filter design for multiple harmonics is not easy or convenient. Moreover, if the frequency range is large, an extremely long filter is required, adding to the computational complexity. An alternative method is to use a target signal whose amplitude and phase are directly pre-defined (shown in Figure 6). This idea avoids the convolution operation with a feedback filter and saves computation cost. Besides, for multiple harmonic controls, it is preferable to use a parallel structure (Kuo and Morgan, 1996) for the independent control of each frequency component. The idea of directly defining the multiple-harmonic target signal is...
extremely convenient for a parallel structure. In the following studies, we will apply it to the MANE for a multiple-harmonic amplitude and phase control (APC) algorithm.

The parallel structure of the multiple-harmonic APC algorithm is shown in Figure 7. Suppose a collection of normalized target frequencies is in the vector

\[ \Omega = \{ \Omega_1, \Omega_2 \} = \{ \omega_i \} = \{ 2\pi f_i / f_s \} \]

\[ = \{ 2\pi f_i / f_s \}^T, \quad (i = 1, 2, \ldots, q = q_1 + q_2) \]  

(29)

where \( \Omega_1 \) contains the frequencies (fundamental and harmonics) of the primary noise, \( \Omega_2 \) contains the frequencies of injection, \( F \) contains the real target frequencies, \( f_s \) is the sampling frequency, \( \omega_i \) and \( f_i \) are the \( i \)th normalized and real target frequency, respectively, and \( q_1 \) and \( q_2 \) are the number of elements in vector \( \Omega_1 \) and \( \Omega_2 \), respectively. Assume that collections of primary amplitudes and primary phases are in the vectors

\[ P_d = \{ p_{di} \}^T, \quad \Phi_d = \{ \phi_{di} \}^T, \quad (i = 1, 2, \ldots, q) \]  

(30)

where \( p_{di} \) and \( \phi_{di} \) are the amplitude and phase of the \( i \)th frequency component of primary noise.

Figure 5. Convergence performance comparison of the active noise equalizer (ANE), normalized active noise equalizer (NANE) and modified active noise equalizer (MANE) (the curves of the NANE and MANE are cut in half for display convenience): (a), (b) and (c) real-error signals when step-sizes are 0.001, 0.002 and 0.005, respectively; (d), (e) and (f) pseudo-error signals when step-sizes are 0.001, 0.002 and 0.005, respectively.

Figure 6. Amplitude and phase control method for active noise and vibration reshaping: (a) filtering the estimation signal of primary noise using an artificially designed filter to obtain the target signal; (b) directly designing the target signal.
The corresponding elements for injection frequencies in \( P_d \) and \( \Phi_d \) are 0. Assume that collections of sinusoidal base signals are in vectors

\[
X_a(n) = [x_{a0}]^T = \cos(\Omega n), \quad X_b(n) = [x_{b0}]^T = \sin(\Omega n)
\]  

Thus, the primary noise \( d(n) \) can be expressed as

\[
d(n) = A_d^T X_a(n) + B_d^T X_b(n),
\]

\[
P_d = \text{diag}(P_d)
\]

where \( A_d \) and \( B_d \) denote the coefficient vectors of the primary noise, and \( \text{diag}[\cdot] \) indicates forming a diagonal matrix using the vector in the bracket. Note that in the following equations, we use the underlined version of the vector variable to denote the diagonal matrix formed by it. The secondary path introduces the amplitude ratio and phase difference to each component. We define them in the following vectors

\[
\Phi = \{\phi_i\}^T, \quad (i = 1, 2, \ldots, q)
\]

From equation (31), the reference signal with amplitude rectification can be expressed as

\[
X_{ar}(n) = P_r X_a(n), \quad X_{br}(n) = P_r X_b(n),
\]

\[
P_r = \text{diag}(P_r), \quad P_r = p_m P_s^{-1}, \quad p_m = \max(P_s)
\]

where \( X_{ar}(n) \) and \( X_{br}(n) \) are the vectors of the rectified reference signal, \( P_r \) contains coefficients for amplitude rectification and \( p_m \) is the maximum amplitude ratio introduced by the secondary path among all the target frequencies. To remain consistent with the ANE, we define gain factors to APC, that is

\[
T = \{\beta_i\}^T, \quad (i = 1, 2, \ldots, q)
\]

where \( \beta_i \) is the gain factor for the \( i \)th component. Thus, the amplitudes of the target signal can be expressed as

\[
P_t = \{p_{ti}\}^T = \tilde{P}_{di}, \quad T = \text{diag}[T].
\]

\[
\tilde{P}_{di} = \{\tilde{p}_{di}\}^T, \quad \tilde{p}_{di} = \begin{cases} p_{di} & \quad p_{di} \neq 0 \quad (i = 1, 2, \ldots, q) \\ p_{di} = 0 \quad (i = 1, 2, \ldots, q) \end{cases}
\]

where \( p_{di} \) is the target amplitude of the \( i \)th component. Since for the injection frequency the corresponding primary amplitudes are zeros, the gain factors of those components are defined as the ratio of \( p_{ti} \) and \( p_{di} \) (the amplitude of the fundamental component of primary noise). APC can also control the phase, so we define the target phase in the vector

\[
\Phi = \{\phi_i\}^T, \quad (i = 1, 2, \ldots, q)
\]

where \( \phi_{ti} \) is the target phase of the \( i \)th component. From equations (36) and (37), the target signal \( t(n) \)
can be expressed as

\[ t(n) = A_n^T X_n(n) + B_n^T X_b(n), \]

\[ A_n = P_n \cos \Phi_n, \quad B_n = P_n \sin \Phi_n, \]

\[ P_n = \text{diag}[P] \]

where \( A_n \) and \( B_n \) are the coefficient vectors of the target signal. From Figure 7, the output of the controller can be expressed as

\[ y(n) = W_a^T(n) X_{aw}(n) + W_b^T(n) X_{bw}(n), \]

\[ W_a(n) = [w_{aw}(n)]^T, \quad W_b(n) = [w_{bw}(n)]^T \]

where \( W_a(n) \) and \( W_b(n) \) denote the coefficient vectors of the controller and \( w_{aw}(n) \) and \( w_{bw}(n) \) are the coefficients of the \( i^{th} \) sub-controller. The residual error is

\[ e(n) = d(n) - y(n) \ast \hat{s}(n) \]

(40)

the estimated primary noise is

\[ \hat{d}(n) = e(n) + y(n) \ast \hat{s}(n) \]

(41)

the estimated error signal is

\[ \hat{e}(n) = \hat{d}(n) - W_a^T(n) [X_{aw}(n) \ast \hat{s}(n)] - W_b^T(n) [X_{bw}(n) \ast \hat{s}(n)] \]

(42)

and the pseudo-error signal is

\[ e_s(n) = \hat{e}(n) - t(n) \]

(43)

From equation (40) and (41), if \( \hat{s}(n) = s(n) \), then the estimated primary noise is equal to the real one, that is

\[ \hat{d}(n) = d(n) \]

(44)

Considering slow adaptation, the position of the controller and the secondary path can be exchanged, so equation (42) can be further express as

\[ \hat{e}(n) = \hat{d}(n) - [W_a^T(n) X_{aw}(n) + W_b^T(n) X_{bw}(n)] \ast \hat{s}(n) \]

(45)

Substituting equation (39) and (44) into equation (45), there is

\[ \hat{e}(n) = d(n) - y(n) \ast s(n) = e(n) \]

(46)

Thus, the pseudo-error signal in equation (43) can also be expressed as

\[ e_s(n) = e(n) - t(n) \]

(47)

The adaptation process of the controller is based on the FXLMS mechanism, that is

\[ W_a(n + 1) = W_a(n) + \mu_e e_s(n) [X_{aw}(n) \ast \hat{s}(n)], \]

\[ W_b(n + 1) = W_b(n) + \mu_e e_s(n) [X_{bw}(n) \ast \hat{s}(n)] \]

(48)

where \( \mu_a \) and \( \mu_b \) are the step-sizes. When the algorithm is convergent, \( e_s \) approaches zero. Thus, from equation (47), the steady-state real-error is

\[ e(n) = t(n) \]

(49)

Since \( t(n) \) is a multiple-harmonic reference signal whose amplitude and phase are predefined, the phase and the amplitude of the real-error are controlled.

APC is derived from the MANE, so it inherits the superiority of convergence property. A comparative study of the ANE and APC is conducted on model equation (27). The simulation parameters are listed in Table 3 and the simulation results are shown in Figure 8. It can be seen that APC has faster convergence in the case of slow tuning (when the step-sizes are 0.001 and 0.002) and a larger stability region in the case of fast tuning (when the step-size is 0.005), so the convergence property is improved compared with the ANE. From equation (49), since \( t(n) \) is the reference signal whose amplitude and phase are predefined, the phase and amplitude of the residual noise can be controlled. The amplitude and phase control capability of APC are shown in Figure 9. The data points shown in the figure are extracted from the real-error signal through fast Fourier transform (FFT). From Table 3, the initial amplitudes (i.e. the mag.s of noise \( d(n) \)) are 1
and the gains are 0.5, so the target amplitude (i.e. the mag.s of target $t(n)$) is $0.5 \times 1 = 0.5$. It can be seen from Figure 9(a) that the amplitude of the real-error signal of both the ANE and APC approach to 0.5 from 1, indicating that both the ANE and APC have the capability of amplitude control. However, as shown in Table 3, only APC can manually define the target phases. As a result, APC has the capability of phase control (from the initial values $90^\circ$ and $100^\circ$ to the target values of $70^\circ$ and $80^\circ$, respectively) as shown in Figure 9(b).
Table 4. Simulation parameters for the demonstration of five control modes of amplitude and phase control (APC).

<table>
<thead>
<tr>
<th>Items</th>
<th>Sampling Freq.(Hz)</th>
<th>Sampling Time (s)</th>
<th>Noise d(n)</th>
<th>Target t(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. (Hz)</td>
<td>Mag.(dB)</td>
<td>Phase (deg.)</td>
<td>Freq. (Hz)</td>
</tr>
<tr>
<td>APC</td>
<td>1024</td>
<td>10</td>
<td>1</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1</td>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1</td>
<td>80</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>1</td>
<td>80</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
</tbody>
</table>

*For the injection mode, the gain is defined as the amplitude ratio of injection and fundamental frequency.
– for a zero target amplitude, the target phase definition makes no sense.

3.2. Noise and vibration injection

Similar to the ANE, APC also has the ability to cancel, suppress, maintain and enhance the primary noises. An additional good property of APC is that it is extremely easy for NVI application: adding parallel branches for the injection frequency components without modifying the structure of the control system. From equation (49), the residual error is equal to the target signal, so if \( t(n) \) contains the frequencies that are beyond \( d(n) \), there will be NVIs. Furthermore, APC can also control the amplitude and phase of the injected frequency components. Therefore, APC actually offers a more general solution to ANVR.

The five control modes (i.e. cancellation, suppression, neutral, enhancement and injection) of APC are tested on the model equation (27). The parameters of the simulation are listed in Table 4. Five frequency components (i.e. 10, 20, 40, 50 and 60 Hz) are considered for the five control modes. Figure 10(a) and (b) show the steady-state spectrum of real-error \( e(n) \) with control off and on, respectively. It can be seen that real-error \( e(n) \) is equal to primary noise \( d(n) \) and target signal \( t(n) \), respectively, when the control is off and on. Since the gain factors for 10, 20, 40 and 50 Hz are 0, 0.5, 1 and 2, respectively, the signals of those components are cancelled, suppressed by 50%, maintained and enhanced by 200%, respectively. For the injection mode, since there is no 60 Hz component in \( d(n) \), the ratio 0.8 means to inject a 60 Hz component whose amplitude is 80% of the fundamental frequency (10 Hz) in \( d(n) \). Figure 10(c) and (d) show the real-time amplitude and phase of the real-error, respectively, when control is on. It can be seen that both the amplitudes and phases are adjusted from those of \( d(n) \) to those of \( t(n) \).

3.3. Convergence analysis

In this section, we will analytically study the convergence property of the proposed APC algorithm.

First, substitute equations (32) and (44) into equation (42), and then substitute equations (42) and (38) into equation (47), then the pseudo-error can be expressed as

\[
e_p(n) = A^T \mathbf{X}_d(n) + B^T \mathbf{X}_b(n) - W_d^T(n) \hat{X}_d(n) - W_b^T(n) \hat{X}_b(n) \tag{50}
\]

where

\[
A = A_d - A_r, \quad B = B_d - B_r \tag{51}
\]

From equations (33) and (34), the filtered reference signal can be expressed as

\[
\hat{\mathbf{X}}_d(n) = p_m \{ \cos \Theta \mathbf{X}_d(n) - \sin \Theta \mathbf{X}_b(n) \}, \quad \hat{\mathbf{X}}_b(n) = p_m \{ \cos \Theta \mathbf{X}_b(n) + \sin \Theta \mathbf{X}_d(n) \} \tag{52}
\]

Substituting equation (52) into equation (50), the pseudo-error can be further expressed as

\[
e_p(n) = \left[ A - \hat{W}_d(n) \right]^T \mathbf{X}_d(n) + \left[ B - \hat{W}_b(n) \right]^T \mathbf{X}_b(n) \tag{53}
\]

where

\[
\hat{W}_d(n) = p_m \{ \cos \Theta_d \cdot W_d(n) + \sin \Theta_d \cdot W_b(n) \}, \quad \hat{W}_b(n) = p_m \{ \cos \Theta_d \cdot W_b(n) - \sin \Theta_d \cdot W_d(n) \} \tag{54}
\]

From equation (53), the expectation of the pseudo-error signal is

\[
E[e_p^2(n)] = \left[ A - \hat{W}_d(n) \right]^T \mathbf{P}_{x,d} \left[ A - \hat{W}_d(n) \right] + \left[ B - \hat{W}_b(n) \right]^T \mathbf{P}_{x,b} \left[ B - \hat{W}_b(n) \right] \tag{55}
\]
where

\[ P_{x,a} = E[X_a(n)X_a^*(n)], \quad P_{x,b} = E[X_b(n)X_b^*(n)] \quad (56) \]

The tuning of the controller coefficient is based on the steepest descent method, that is

\[
\begin{align*}
W_a(n + 1) &= W_a(n) + \mu_a \frac{\partial E[e_s^2]}{\partial W_a(n)}, \\
W_b(n + 1) &= W_b(n) + \mu_b \frac{\partial E[e_s^2]}{\partial W_b(n)}
\end{align*}
\]  

(57)

where the gradients can be obtained by substituting equation (54) into equation (55), that is

\[
\frac{\partial E[e_s^2]}{\partial W_a(n)} = -2p_m^2 P_{x,a} W_a(n)
\]

+ 2p_m P_{x,a} [\cos \Phi_a \cdot A - \sin \Phi_a \cdot B]

\[
\frac{\partial E[e_s^2]}{\partial W_b(n)} = -2p_m^2 P_{x,b} W_b(n)
\]

(58)

Substituting equation (58) into equation (57), the coefficients updating equations can be further expressed as

\[
\begin{align*}
W_a(n + 1) &= K_a W_a(n) + M_a, \\
W_b(n + 1) &= K_b W_b(n) + M_b
\end{align*}
\]  

(59)

where

\[
\begin{align*}
K_a &= I - \mu_a p_m^2 P_{x,a}, \\
K_b &= I - \mu_b p_m^2 P_{x,b}, \\
M_a &= \mu_a p_m P_{x,a} [\cos \Phi_a \cdot A - \sin \Phi_a \cdot B], \\
M_b &= \mu_b p_m P_{x,b} [\sin \Phi_a \cdot A + \cos \Phi_a \cdot B]
\end{align*}
\]

(60)

The convergence equations of the coefficients are

\[
\begin{align*}
W_a(n) &= K_a^n W_a(0) + (I - K_a)^{-1} M_a (I - K_a^{n-1}), \\
W_b(n) &= K_b^n W_b(0) + (I - K_b)^{-1} M_b (I - K_b^{n-1})
\end{align*}
\]  

(61)
From equation (61), if $\|K_a\|_\infty < 1$ and $\|K_b\|_\infty < 1$, the steady-state coefficients of the controller are

$$W_a(\infty) = (1 - K_a)^{-1}M_a = p_m^{-1}[\cos \Phi_a \cdot A - \sin \Phi_a \cdot B],$$
$$W_b(\infty) = (1 - K_b)^{-1}M_b = p_m^{-1}[\sin \Phi_a \cdot A + \cos \Phi_a \cdot B]$$

(62)

and the step-size bound is

$$0 < \mu_a, \mu_b < \frac{2}{p_m^2 \|P_{x,a}\|_\infty} = \frac{2}{p_m^2 \|P_{x,b}\|_\infty}$$

(63)

From equations (31) and (56), $\|P_{x,a}\|_\infty = \|P_{x,b}\|_\infty = 1/2$. equation (63) is obtained according to the mean-square pseudo-error. However, in real applications, the instantaneous-square pseudo-error is applied for the coefficient updating. The maximum-square (instead of mean-square) value of the pseudo-error will be more practical. From equation (31), we define

$$P_{x,a} = \text{diag}[\max[x^2_a(n)]]^T, \quad P_{x,b} = \text{diag}[\max[x^2_b(n)]]^T$$

(64)

Thus, the practical step-size bound is

$$0 < \mu_a, \mu_b < \frac{2}{p_m^2 \|P_{x,a}\|_\infty} = \frac{2}{p_m^2 \|P_{x,b}\|_\infty}$$

(65)

where $\|P_{x,a}\|_\infty = \|P_{x,b}\|_\infty = 1$. It can be seen that equation (65) is equivalent to equation (23), which means that APC has the same transient property as the MANE.

Next, we will deduce the amplitude and phase of the real-error signal. From equation (47), there is

$$e(n) = e_r(n) + t(n)$$

(66)

Substituting equations (53), (51) and (38) into equation (66), the real-error is

$$e(n) = \left[ A - \hat{W}_a(n) \right]^T X_a(n) + \left[ B - \hat{W}_b(n) \right]^T X_b(n)$$
$$+ A_{\hat{t}}^T X_a(n) + B_{\hat{t}}^T X_b(n)$$
$$= \left[ A_{\hat{t}} - \hat{W}_a^T(n) \right] X_a(n) + \left[ B_{\hat{t}} - \hat{W}_b^T(n) \right] X_b(n)$$

(67)

So the amplitude and phase of the real-error are

$$P_e(n) = \sqrt{[A_{\hat{t}} - \hat{W}_a(n)]^2 + [B_{\hat{t}} - \hat{W}_b(n)]^2}$$

(68)

Therefore, the theoretical transient (real-time) amplitude and phase can be obtained by equations (68), (54) and (59). The theoretical steady-state amplitude and phase can be obtained by equations (68), (54) and (62).

A simulation is conducted on the secondary path model, as shown in equation (27), to verify the theoretical analysis. The parameters of the simulation are listed in Table 5. The convergence processes of the controller coefficients are shown in Figure 11. The primary noise contains two components, 10 and 20 Hz, and each component requires two coefficients, $w_a(n)$ and $w_b(n)$. The theory asymptotes of each component are calculated by equation (62); the theory curves of each component are calculated by equation (59); the simulation curves of each component are collected in the simulation process (as shown in equation (48)).

It can be seen that the theory curves and simulation curves are mismatching at the very beginning. This is because the theoretical analyses are based on the statistical index (i.e. the mean-square value) of the real-error signal and reference signals, while the simulations are based on the transient value of those signals. In any case, the theory curves and simulation curves finally approach the same asymptotes, which verifies the validity of the theoretical analysis. The real-time amplitudes and phases of each component of the real-error are shown in Figure 12, where Figure 12(a) shows the amplitude curves of the real-error signal and Figure 12(b) shows the phase curves of the real-error signal. The theory curves are calculated by equation (68); the simulation curves (both amplitude curves and phase curves) are extracted from the simulation result of the FFT. Similarly, although the theory curves and simulation curves are mismatched at the beginning, they finally approach the same target value, which can also verify the validity of the theoretical analysis.
Figure 11. Controller coefficients, a comparison of theory and simulation: (a) coefficients of the sine branch; (b) coefficients of the cosine branch.

Figure 12. Real-time amplitude and phase of the real-error, a comparison of theory analysis and simulation: (a) real-time amplitude of each component of the real-error; (b) real-time phase of each component of the real-error.

Table 6. Summary of some of the algorithms for active noise and vibration reshaping.

<table>
<thead>
<tr>
<th>Item</th>
<th>ANE</th>
<th>NANE</th>
<th>MANE</th>
<th>APC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convergence rate</td>
<td>Slow</td>
<td>Fast</td>
<td>Fast</td>
<td>Fast</td>
</tr>
<tr>
<td>Stability region</td>
<td>Small</td>
<td>Small</td>
<td>Large</td>
<td>Large</td>
</tr>
<tr>
<td>Amplitude control ability</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Phase control ability</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Noise and vibration modifying ability (i.e. cancellation, suppression, neutral and enhancement)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Noise and vibration injection (NVI) ability</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

ANE: active noise equalizer; NANE: normalized active noise equalizer; MANE: modified active noise equalizer; APC: amplitude and phase control.
3.4. Discussion

Table 6 provides a rough comparison of the various algorithms for ANVR that are considered in this paper, that is, the ANE, the NANE, the MANE, and APC. The comparison includes convergence, the stability region, the amplitude control ability, the phase control ability, the noise/vibration modifying ability (i.e. cancel, suppress, maintain or enhance the existing frequency components in the primary noise) and the noise/vibration injection ability (i.e. inject the new frequency components to the system). It can be seen that APC has superiority in almost all the items, especially for the phase control ability and NVI ability. APC possess a larger stability region, since we apply the modified FXLMS structure.

APC has a uniform convergence rate in the application of multiple-harmonic control, since the uneven amplitudes introduced by the secondary path model are compensated by reference rectification. Most importantly, the advantages and applications of phase control and NVI are specially emphasized in this paper.

4. Case study using experimental data

In this section, case studies are conducted using experimental data. The multiple-harmonic primary noise was...
measured from a rotor test-platform. The data sets used for the secondary path identification were measured from a random vibration test of a clamped-free shell structure. The identification system, consisting of an actuator, amplifier, PC, conditioner, accelerometer, etc., is shown in Figure 13. According to the excitation signal generated by the PC and the response signal measured by the PC, the transfer function from A to

![Figure 15](image-url)

**Figure 15.** Unit impulse response and frequency response functions (FRFs): (a) full-length unit impulse response signal; (b) truncated unit impulse response signal; (c) and (d) amplitude-frequency and phase-frequency diagram, respectively, of the averaged, truncated and identified FRFs. Figure 16. Experimental primary noise measured from a rotor test-platform: (a) the time domain signal; (b) the spectrum of the primary noise.

![Figure 16](image-url)

**Figure 16.** Experimental primary noise measured from a rotor test-platform: (a) is timedomain588 signal; (b) is spectrum of the primary noise.
B can be identified. Since the sensitivity of the accelerometer is known (100 mV/g) and the conditioner has a unity gain, we display the measured signal in the form of acceleration (g) rather than voltage (V). Thus, the unit of the following transfer functions will be g/V.

4.1. System identification

Broad-band excitation and the corresponding response signals can be applied for the system identification. Multiple sets of excitation/response signals can be applied for the system identification.

---

Table 7. Parameters for the study of amplitude and phase control based on experimental data.

<table>
<thead>
<tr>
<th>No.</th>
<th>Freq.(Hz)</th>
<th>Noise d(n)</th>
<th>Phase (deg.)</th>
<th>Target mag.(mm)</th>
<th>Target phase(deg.)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Case A</td>
<td>Case B</td>
<td>Case A</td>
</tr>
<tr>
<td>1</td>
<td>38.609</td>
<td>0.0087</td>
<td>88.00</td>
<td>0.0087</td>
<td>0.0000</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>77.220</td>
<td>0.0113</td>
<td>57.63</td>
<td>0.0113</td>
<td>0.0068</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>115.83</td>
<td>0.0034</td>
<td>89.81</td>
<td>0.0034</td>
<td>0.0034</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>154.44</td>
<td>0.0019</td>
<td>111.1</td>
<td>0.0019</td>
<td>0.0056</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>193.05</td>
<td>0.0012</td>
<td>154.3</td>
<td>0.0012</td>
<td>0.0046</td>
<td>90</td>
</tr>
<tr>
<td>6</td>
<td>231.65</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0052</td>
<td>0.0078</td>
<td>160</td>
</tr>
</tbody>
</table>

*For the injection mode, the gain is defined as the amplitude ratio of injection and fundamental frequency — for a zero target amplitude, the target phase definition makes no sense.

---

Figure 17. Control results using experimental data, case A: (a) and (b) the real-error signal in the time periods of 0–0.1 and 6.6–6.7 s, respectively, with both control off and on; (c) steady-state spectrum of the real-error signal with control off and on; (d) and (e) real-time amplitude and phase, respectively, of each component of the real-error when control is on.
used to improve the identification accuracy. In this study, 20 sets of excitation/response data were acquired and a high-order infinite impulse response (IIR) filter (both 100 backward and forward coefficients) was applied for the system identification according to the above procedure. Figure 14(a) and (b) show the random excitation and response of the 20th set of data, respectively. Figure 14(c) and (d) show the amplitude-frequency and phase-frequency diagrams, respectively, of the estimated frequency response functions (FRFs) using both one set of data and an average of the 20 sets of data. It can be seen that the FRF estimated by multiple sets of data (averaged FRF) has a better signal-to-noise ratio (SNR). Figure 15(a) shows the impulse response signal estimated from the averaged FRF. It can be seen that after 0.3 s, the impulse response decays nearly to zero and the residual signals are mostly noise. Thus, we truncate the impulse response signal to save computation cost as well as reduce noise (as shown in Figure 15(b)). Figure 15(c) and (d) show the amplitude-frequency and phase-frequency diagrams, respectively, of the averaged, truncated and identified FRFs. It can be seen that (1) a much better SNR can be achieved by using the truncated impulse response; (2) the IIR filter can precisely identify the secondary path.

4.2. Control simulation

Case studies have been conducted based on the experimental primary noise and the above-identified secondary path model (IIR filter). The primary noise collected from the rotor test-platform is shown in Figure 16. The sampling frequency is 3000 Hz and the sampling time is 6.83 s. The primary noise contains five prominent components whose frequencies are 38.609 Hz (1×), 77.220 Hz (2×), 115.83 Hz (3×), 154.44 Hz (4×) and 193.05 Hz (5×). The amplitudes of each component are ~0.0087, ~0.0113, ~0.0034, ~0.0019 and ~0.0012 mm. The phases of each component are ~88.00°, ~57.63°, ~89.81°, ~111.1° and ~154.3°. Two cases are considered by defining different target amplitudes and...
phases. The parameters of the case studies are listed in Table 7.

4.2.1. Case A. In case A, we focus on the phase control capability of APC. The amplitude ratios of all the components are 1, that is, the amplitudes of the target signal are the same as that of the primary noise, as shown in Table 7. In particular, for the injection mode (component 6×), the ratio is 0.6, which means that the target magnitude is 60% of the fundamental component (1×) of the primary noise. The target phases of each component can also be found in Table 7. The control results are shown in Figure 17. Figure 17(a) and (b) show the real-error signal with control on and off in different time periods: (a) is 0–0.1 s and (b) is 6.6–6.7 s. It can be seen that (1) the real-error is equal to the primary noise when control is off; (2) the real-error is equal to the primary noise at the very beginning when control is on; after a period of time with control on, there are phase shifts while the amplitude remains somewhat the same. Figure 17(c) shows the steady-state spectrum of the real-error with control on and off. It can be further seen that (1) the amplitudes of components 1×–5× are the same when control is on and off; (2) there is a noise and vibration injection (6×) at 231.65 Hz. Figure 17(d) and (e) show the convergence process of amplitude and phase, respectively, of each component of the real-error. It can be seen that (1) the amplitude of each component is indeed kept the same (around the dotted-line); (2) the phases approaches the targets (the dotted-line) from the initial values.

4.2.2. Case B. In case B, we carry out a more general simulation of APC to test its five working modes. The 618 amplitude ratios are 0, 0.6, 1, 3, 4 and 0.9, indicating to cancel the 1× component, suppress the 2× component by 60%, maintain the 3× component, enhance the 4× component by 200%, enhance the 5× component by 400% and inject a 6× component whose amplitude is 90% of the fundamental frequency (1×), respectively. The control results are shown in Figure 18. Figure 18(a) and (b) also show the real-error signal with control on and off in different time periods: (a) is 0–0.1 s and (b) is 6.6–6.7 s. It can be seen from Figure 18(a) that the real-error is equal to the primary noise at the very beginning when the control is on. After a period of time with control on, both phase and amplitude (the shape of the wave) are changed (Figure 18(b)). A more clear illustration is the steady-state amplitude spectrum with control on and off, as shown in Figure 18(c). It can be seen that the five control modes (cancellation, suppression, neutral, enhancement and injection) have been successfully achieved. Figure 18(d) and (e) show the amplitude and phase convergence process, respectively. It can be seen that both the amplitude and phase approach their targets (the dotted-line), including the injection component.

5. Conclusions

In this paper, an APC algorithm for ANVR has been proposed. It has two critical functions: (1) control the amplitude and phase of the primary noise; (2) inject the frequency component that is not included in the primary noise.

Two improvements have been made for a better convergence property: (1) the modified FXLMS structure has been applied for a larger stability region; (2) the influence of secondary path model on the reference signal is compensated to obtain uniform convergence rate among the different frequency components.

Analytical investigation of APC has been conducted and the step-size bounds of APC are provided. The results have been compared to those of simulations, which verify the validity of the proposed APC algorithm.

Control simulation using experimental data has also been carried out. The primary noise is a multiple-harmonic signal measured from a rotor test-platform. The secondary path model is identified according to a random vibration test on a clamped–free shell structure. The results verify the robustness of the proposed APC algorithm. All in all, APC offers a more general and effective solution to ANVR, which, we believe, will enrich the applications of active noise and vibration control.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the National Natural Science Foundation of China (nos. 51405370, 51421004), the National Key Basic Research Program of China (no. 2015CB057400) and the China Postdoctoral Science Foundation (nos. 2016T90908, 2016MS90937).

References


