Observation of triple-dressing on photonic band gap of optically driven hot atoms

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We experimentally investigate transmission of the probe field, the reflected four-wave mixing photonic band gap signal, and fluorescence signal in a five-level atomic system. In addition, we compared the single-dressing, double-dressing, and triple-dressing on the three types of signals. These three types of signals can be controlled by frequency detunings, powers, and relative phases of the dressing beams. Such a scheme could have potential applications in amplification processing of triode and quantum information processing.

1. INTRODUCTION

Four-wave mixing (FWM) is a well-known nonlinear optical effect [1] that can be enhanced in an electromagnetically induced transparency (EIT) medium [2–4]. The EIT-based nonlinear schemes can be driven by traveling wave beams [5–9] and by a standing wave (SW) because of two counter propagating coupling fields [10–12]. Interaction of the SW with the atomic coherent medium results in an electromagnetically induced grating (EIG) [13–17], which possesses photonic band gap (PBG) structure, has a potential application in all optical switches [18], and provides manipulation of light propagation to create a tunable photonic band gap [19,20]. Additionally, the optical response of the ultra cold $^{85}$Rb atoms driven by a stationary SW coupling field, both to a static and to a time-dependent probe field, was investigated [21]. In addition, the fluorescence (FL) signal induced by spontaneous emission under EIT conditions has been studied in lithium molecules [22,23]. Intensities of these three types of signals satisfy the law of conservation of energy [24].

In our paper, we first investigated the photonic band gap (PBG) structure dressed by two external-dressing fields ($E_3$ and $E_5$) and self-dressing fields ($E_3$ and $E_5'$) in hot vapor of $^{85}$Rb atoms. By optically controllable PBG structure, we experimentally observe not only the transmission of the probe field, the reflected four-wave mixing photonic band gap signal (FWM BGS), and the FL signal, but also the enhancement and suppression of these signals under the triple-dressing effect in a five-level system by controlling the frequency detuning, powers, and relative phases of the dressing beams. Moreover, the transmission and reflection efficiency in a single-dressing field, a double-dressing field, and a triple-dressing field are compared. Furthermore, triple-dressing effects on the photonic band structure at different conditions are investigated, which can modify the reflection efficiency. Interplay of the two doubly dressed fields is observed to deeply understand the dressing effect on the PBG structure.

2. EXPERIMENTAL SETUP AND BASIC THEORY

In this paper, we use a five-level rubidium ($^{85}$Rb) atomic system composed by $5S_{1/2}(F=3)(|0\rangle)$, $5S_{1/2}(F=2)(|3\rangle)$, $5P_{3/2}(|3\rangle)$, $5D_{3/2}(|4\rangle)$, and $5D_{5/2}(|2\rangle)$. Rubidium atomic vapors were contained in a cylindrical glass cell (length 8.0 cm, width 6.0 cm, and diameter D 2.0 cm), as shown in Fig. 1(a). A probe transmission signal (PTS), FWM BGS, and three FL signals exist under the normal experimental configuration. Specifically, the FWM BGS is generated by $E_1$, $E_5$, and $E_5'$, and satisfies the phase-matching condition $k_F = k_1 + k_3 + k_5'$. Five laser beams generated by four external cavity diode lasers (ECDL) with linewidths of less than or equal to 1 MHz are used here. Coupling laser beams $E_3$ (frequency $\omega_3$, wave vector $k_3$, and Rabi frequency $G_3$), and $E_5'$ ($\omega_3$, $k_3'$, and $G_3'$) connect the transition $|3\rangle \rightarrow |1\rangle$ with a wavelength about 780.238 nm;
and 8.4 mW, respectively. In addition, \( k_\text{E} \) and \( \Delta_0 \) angle, as shown in Fig. 1(c). The generated FWM BGS (d4) and the velocity \( v \) of the electric field, the frequency detuning \( \Delta_1 = \omega_1 - \omega_2 \) will form an SW. The EIG will lead to a phase-matching condition \( k_F = k_1 - k_2 + k_3 \). (e) Dressed state

\[
\rho^{(1)}_{10} = \frac{iG_1}{d_{10} + |G_{31}|^2/d_{30} + |G_{21}|^2/d_{21} + |G_{41}|^2/d_{41}},
\]

and

\[
\rho^{(3)}_{10} = \frac{-iG_1G_2G_3'}{d_{10} + |G_{31}|^2/d_{30} + |G_{21}|^2/d_{21} + |G_{41}|^2/d_{41}}d_{30},
\]

where \( d_{10} = \Gamma_{10} + i(\Delta_1 - k_1v) \), \( d_{30} = \Gamma_{30} + i\Delta_1 - i\Delta_3 = \Gamma_{30} + i(\Delta_1 - \Delta_3 - (k_3v - k_3v)) \), \( d_{21} = \Gamma_{20} + i\Delta_1 + i\Delta_2 = \Gamma_{20} + i(\Delta_1 + \Delta_2 - (k_1v - k_1v)) \), \( d_{41} = \Gamma_{40} + i\Delta_1 + i\Delta_4 = \Gamma_{40} + i(\Delta_1 + \Delta_4 - (k_3v - k_3v)) \), \( \Delta_1 = \Omega_{10} - \omega_1 \), \( \Omega_1 \) is the resonance frequency of the transition driven by \( E_1 \), and \( \Gamma_1 \) is the transverse relaxation rate between \( i \) and \( j \). Because of the three resonant window (\( \Delta_1 - \Delta_3 - (k_3v - k_3v), \Delta_1 + \Delta_2 - (k_1v - k_1v) \), and \( \Delta_1 + \Delta_4 - (k_3v - k_3v) \)), the residual Doppler is dominant in these three resonant points.

According to the relation \( d_\alpha d E = N_\alpha \int_\infty^\infty d\omega \rho_\omega (\omega) \), where \( \rho_\omega (\omega) = 1/(\pi \mu_\omega)^{1/2} \exp(-\omega^2/\mu_\omega^2) \), \( N_\alpha = P_\alpha / kT \) is the density of the atoms (cm\(^{-3}\)), \( P_\alpha \) is the vapor pressure of the cell (Torr), \( T \) is the temperature of the cell (K), and \( k = 1.38065 \times 10^{-23} \) is the Boltzmann constant, we can get the formulas of susceptibilities as follows:

\[
\chi^{(1)} = \frac{iN\mu^2}{\hbar \epsilon_0 d_{10} + |G_{31}|^2/d_{30} + |G_{21}|^2/d_{21} + |G_{41}|^2/d_{41}}
\]

and

\[
\chi^{(3)} = \frac{1}{-iN\mu^2} \frac{iN\mu^2}{\hbar \epsilon_0 d_{10} + |G_{31}|^2/d_{30} + |G_{21}|^2/d_{21} + |G_{41}|^2/d_{41}} d_{30}
\]

The nonlinear coupled wave equations [25], \( \partial E_1(x)/\partial x = -\alpha E_1(x) + k \exp(-i\Delta k x) \) and \( \partial E_F(x)/\partial x = -\alpha E_F(x) + k \exp(i\Delta k x) E_1(x) \), are given to estimate the reflectivity efficiency, where \( E_1(x) \) and \( E_F(x) \) stand for the probe and FWM BGS field, respectively; \( \alpha = (\omega_1/c) \Im \chi^{(1)} / 2 \) is the attenuation of the field because of the absorption of the medium; \( k = i(\omega_1/c) \Im \chi^{(3)} / 2 \) is the gain because of the nonlinear susceptibility; and \( \chi^{(1)} \) and \( \chi^{(3)} \) are the zero order coefficients from Fourier expansion of \( \chi^{(1)} \) and \( \chi^{(3)} \), respectively. \( \Delta k = [2(\omega_1 \cos \theta - \omega_3) + \Re \chi^{(3)}(\omega_1 \cos \theta)] / \cos \phi \) is the phase mismatch magnitude, in which \( \phi \) is the angle between probe \( E_1 \) and \( E'_1 \). By solving the above equations, the reflected FWM BGS (R) and PTS (T) can be obtained as

\[
R = \frac{1}{k (\lambda_1^2 + \alpha)^{1/2}} \exp(-\lambda_2^2 dx) (\lambda_1 + \alpha)^{-1} \exp(-\lambda_2^2 dx)
\]

and

\[
T = \frac{\exp(\lambda_1^2 dx) (\lambda_1 - \lambda_1^2 dx)^{1/2}}{\exp(\lambda_1^2 dx) (\lambda_1^2 + \alpha)^{-1} \exp(\lambda_1^2 dx).}
\]
where \( d_x \) is the length of the sample in the x direction. \( \lambda_1^\pm = -i\Delta_k f / 2 \pm [(\alpha - i\Delta f / 2)^2 - \kappa^2]^{1/2} \) and \( \lambda_2^\pm = \lambda_1^\pm + i\Delta f \).

For the FL signal, the second-order FL signal caused by the photon decay from the level [1] to [0] with a wavelength of 780 nm is described by \( \rho_{00}^{(2)} \rightarrow \rho_{10}^{(2)} \rightarrow \rho_{11}^{(2)} \). By solving the coupled density matrix equations, expression of the density matrix element \( \rho_{11}^{(2)} \) can be written as \( \rho_{11}^{(2)} = -[G_1^2 / d_{10} \Gamma_{11}] \), the amplitude squared of which is proportional to the intensity of FL signal when the beams \( E_2 \) and \( E_4 \) are turned on, the FL process is dressed, and the expression of \( \rho_{11}^{(2)} \) can be modified as

\[
\rho_{11}^{(2)} = \frac{-[G_1^2]}{\Gamma_{11}(d_{10} + |G_1|^2/d_{10} + |G_2|^2/d_{21} + |G_4|^2/d_{41})}. \tag{3}
\]

The fourth-order FL signal is described by the photon decay from the level [2] to [1] and [4] to [1] both with a wavelength of 776 nm, respectively.

Via the Liouville pathway, \( \rho_{00}^{(4)} \rightarrow \rho_{10}^{(4)} \rightarrow \rho_{20}^{(4)} \rightarrow \rho_{11}^{(4)} \rightarrow \rho_{22}^{(4)} \), we can obtain the density matrix element of fourth-order FL signal as

\[
\rho_{22}^{(4)} = (|G_1|^2 |G_2|^2) / \Gamma_{22}(d_{10} + |G_2|^2/d_{21} + |G_4|^2/d_{41} + |G_1|^2/d_{31}), \tag{4}
\]

where \( d_3 = \Gamma_{31} + i\Delta_2 \). The dressing effect should be considered, and the dressed FL signal is given as

\[
\rho_{22}^{(4)} = \frac{|G_1|^2 |G_2|^2}{\Gamma_{22}(d_{10} + |G_4|^2/d_{41} + |G_2|^2/d_{21} + |G_1|^2/d_{31})}. \tag{5}
\]

Similarly, for fourth-order FL via the Liouville pathway, \( \rho_{00}^{(4)} \rightarrow \rho_{10}^{(4)} \rightarrow \rho_{40}^{(4)} \rightarrow \rho_{41}^{(4)} \rightarrow \rho_{44}^{(4)} \), we can obtain the density matrix element \( \rho_{44}^{(4)} \) as \( \rho_{44}^{(4)} = (|G_1|^2 |G_4|^2) / \Gamma_{44}(d_{10} + |G_4|^2/d_{41} + |G_2|^2/d_{21} + |G_1|^2/d_{31}) \)

where \( d_6 = \Gamma_{41} + i\Delta_4 \). When the beam \( E_2 \) is turned on, its dressing effect should be considered and the doubly dressed FL signal is given as

\[
\rho_{44}^{(4)} = \frac{|G_1|^2 |G_4|^2}{\Gamma_{44}(d_{10} + |G_2|^2/d_{21} + |G_4|^2/d_{41} + |G_1|^2/d_{31})}. \tag{5}
\]

We know that the essential requirement to attain the phenomenon of PBG is to have a medium in which the refractive index is periodic. According to the relation of refractive index with susceptibility, i.e., \( n = (1 + \text{Re}(\chi))^{1/2} \), the susceptibility should be periodic to get the periodic refractive index. Further, we should gain the periodic split energy level structure for the purpose of getting the periodic susceptibility. Hence, the first step is to gain the periodic split energy level structure.

As shown in Fig. 1(d1), affected by \( |G_3| \), the level [1] will be split into two dressed states expressed as \( |G_3\pm| \), which are periodic along the x direction because of the periodic \( |G_3|^2 \) in the x direction. Eigenvalues of the two dressed states are \( \lambda_{|G_3\pm|} = -\Delta_3 f / 2 \pm (\Delta_3^2 f / 2 + |G_3|^2)^{1/2} \). For the double-dressed case, as shown in Fig. 1(d2), \( E_2 \) acts as one dressing field, so the first-level dressed states \( |G_3\pm| \) will be split into second-level dressed states, \( |G_3 - G_2\pm| \) and \( |G_3 + G_2\pm| \), with eigenvalues

\[
\lambda_{|G_3 - G_2\pm|} = \left[ -\Delta_3 - (\Delta_3^2 f / 2 + |G_3|^2)^{1/2} \right] / 2 + \left[ |\Delta_3| \pm (\Delta_3^2 f / 2 + |G_3|^2)^{1/2} \right] / 2 \]

and

\[
\lambda_{|G_3 + G_2\pm|} = \left[ -\Delta_3 - (\Delta_3^2 f / 2 + |G_3|^2)^{1/2} \right] / 2 + \left[ |\Delta_3| \pm (\Delta_3^2 f / 2 + |G_3|^2)^{1/2} \right] / 2,
\]

where \( \Delta_3 = \Delta_3 - [\Delta_3 - (\Delta_3^2 f / 2 + |G_3|^2)^{1/2}] / 2 \) and \( \Delta_3^2 = \Delta_3 - [\Delta_3 - (\Delta_3^2 f / 2 + |G_3|^2)^{1/2}] / 2 \). For the triple-dressed case in Fig. 1(d3), when \( E_4 \) is turned on, second-level dressed states \( |G_3 \pm G_2 \pm| \) are further split into third-level dressed states \( |G_3 \pm G_2 \pm G_4\pm| \) with eigenvalues

\[
\lambda_{|G_3 + G_2 + G_4\pm|} = \left[ -\Delta_3 - (\Delta_3^2 f / 2 + |G_3|^2)^{1/2} \right] / 2 + \left[ |\Delta_3| \pm (\Delta_3^2 f / 2 + |G_3|^2)^{1/2} \right] / 2
\]

and

\[
\lambda_{|G_3 + G_2 + G_4\pm|} = \left[ -\Delta_3 - (\Delta_3^2 f / 2 + |G_3|^2)^{1/2} \right] / 2 + \left[ |\Delta_3| \pm (\Delta_3^2 f / 2 + |G_3|^2)^{1/2} \right] / 2.
\]

where \( \Delta_4 = \Delta_4 - [\Delta_4 - (\Delta_4^2 f / 2 + |G_4|^2)^{1/2}] / 2 \) \( \text{and} \) \( \Delta_4^2 = \Delta_4 - [\Delta_4 - (\Delta_4^2 f / 2 + |G_4|^2)^{1/2}] / 2 \). The \( |G_3 \pm G_2 \pm G_4\pm| \) are all periodic along the x direction, which induced periodic susceptibility \( \chi^{(3)} \). The corresponding spatial periodic energy levels are in Figs. 1(d4)–1(d6). Figures 1(e1)–1(e5) show the triple-dressed state picture. The third-level dressing states \( |G_3 \pm G_2 \pm G_4\pm| \) will move with changing \( \Delta_4 \). In Fig. 1(e3), because of the three photon resonances with \( \Delta_4 = -\Delta_2 = -\Delta_4 \), only three dressed states (solid lines) appear.

### 3. RESULTS AND DISCUSSIONS

#### A. Different Dressing Results Versus Probe Detuning

First, we observed the PTS, FWM BGS, and FL signals when we scanned the probe frequency detuning \( \Delta_1 \) with different beams blocked with \( N = 5.603 \times 10^{11} \) m\(^{-3}\). Figures 2(a1)–2(a4) show the PTS. There is a Doppler absorption background at the single-photon resonant condition \( \Delta_1 = 0 \).
because of the term $d_{10} = \Gamma_{10} + i(\Delta_1 - k_1 v)$ and an inverted enhancement peak, i.e., the PTS related to $T$ of Eq. (1). When we block $E_2$ and $E_4$ in Fig. 2(a1), the single-dressed PTS generated by $E_1$ is only dressed by $E_3$ ($E_3^*$). The intensity of the PTS also reaches smallest without the dressing of the extra dressing field $E_1$ or $E_2$. With the $E_4$ (or $E_2$) beam on, the energy level can be further dressed [Fig. 1(d2)], and the PBG structure will be influenced so that the dressed PTS increases because of the double-dressing effect of $E_4$ (or $E_2$) with $E_3$ ($E_3^*$) at the resonate point $\Delta_1 = \Delta_3 = -\Delta_2$ ($\Delta_1 = \Delta_3 = -\Delta_4$), according to the dressing term $|G_4|^2/d_{41}$ (or $|G_3|^2/d_{31}$) in Eq. (1), as shown in Fig. 2(a2) [or Fig. 2(a3)]. As $P_2 > P_4$, the dressed PTS in Fig. 2(a3) is larger than that in Fig. 2(a2) because $G_2$ is stronger than $G_4$. With all five beams on, the PTS reaches maximum at $\Delta_1 = \Delta_3 = -\Delta_2 = -\Delta_4$ because of the triple-dressing effect of $E_2$, $E_4$ and $E_3$ ($E_3^*$), as shown in Fig. 2(a4). In Fig. 2(b1), the emission peak is FWM BGS, represented by $R$ in Eq. (2), which is from a reflection of the PBG structure. Without the dressing fields $E_1$ or $E_2$, the emission peak reaches its highest value. With $E_4$ (or $E_2$) on, the FWM BGS will be suppressed by dressing field $E_1$ (or $E_2$), according to $|G_4|^2/d_{41}$ (or $|G_3|^2/d_{31}$) in $\rho_{10}^{(3)}$, as shown in Fig. 2(b2) [or Fig. 2(b3)]. Because of the strongest suppression by both $E_4$ and $E_2$, the FWM BGS reaches a minimum with $E_4$ and $E_2$ both on, as shown in Fig. 2(b4). For the FL signal, Fig. 2(c1) represents the second-order FL gadgets $\rho_{21}^{(2)}$ induced by $E_1$, $E_3$ and ($E_3^*$), with $E_3$ and $E_1$ both blocked. The small dip is induced by dressing field $E_3$ ($E_3^*$) at the resonate point $\Delta_1 - \Delta_3 = 0$, according to the term $|G_3|^2/d_{30}$ in Eq. (3). When we open $E_4$ (or $E_2$), a small, sharp emission peak appears at the resonate point $\Delta_1 = \Delta_3 = -\Delta_4$ (i.e., $-\Delta_2$) on the FL gadgets. It is four-order FL gadgets (FL Gadgets), as shown in Fig. 2(c2) [or Fig. 2(c3)], and the corresponding equation of the signal is $\rho_{44}^{(4)}$ (or $\rho_{22}^{(4)}$). In Fig. 2(c4), the emission peak reaches maximum with all five beams on because of the overlap of the FL gadgets and FL gadgets signals.

**B. Detuning Dependence**

In the following, we observe the double-dressed and triple-dressed signals by scanning $\Delta_1$ at different $\Delta_1$ (or $\Delta_3$). Figures 3(a1)–3(c1) are the double-dressed signals with scanning $\Delta_4$ at different discrete $\Delta_1$, without the dressing field $E_2$. In Fig. 3(a1), the profile (dashed curve) of these PTS consists of baselines shows a Doppler absorption background with a peak (named profile peak) on it. The profile peak in Fig. 3(a1) is the intensity of single-dressed PTS induced by $E_3$ ($E_3^*$) at $\Delta_1 = \Delta_3$ because of the term $|G_3|^2/d_{30}$ of $\rho_{10}^{(3)}$ in Eq. (1). The peaks in each sub-curve which are double-dressed signals with scanning $\Delta_1$ at certain $\Delta_1$ stand for the enhanced PTS induced by $G_4$ at $\Delta_1 = -\Delta_1$, according to the dressing term $|G_4|^2/d_{41}$ in Eq. (1). It reaches the highest at $\Delta_1 = 0$ because of the smallest photonic absorption. The total intensity of PTS reaches maximum at $\Delta_4 = -\Delta_1 = -\Delta_3$. In Fig. 3(b1), the profile (dashed curve) consisting of baselines shows the FWM BGS introduced by $E_1$, $E_3$, and $E_3$. A dip in each sub-curve represents the suppression of the FWM BGS because of the dressing effect of $E_4$ meeting the condition $\Delta_4 = -\Delta_1$, corresponding to the dressed PTS peak because of the term $|G_4|^2/d_{41}$ in Eq. (2). The deepest dip appears at $\Delta_4 = -\Delta_1 = -\Delta_3$, corresponding to the strongest PTS. In Fig. 3(c1), the profile (dashed curve) consisting of baselines of the FL signal is the FL signal induced by $E_3$ ($E_3^*$), and it reaches minimum at $\Delta_1 - \Delta_3 = 0$ because of the term $|G_3|^2/d_{30}$ in Eq. (3). In each sub-curve, the peak is a fourth-order FL gadgets signal related to $\rho_{44}^{(4)}$, and it reaches its smallest value at $\Delta_4 = 0$ because of the strongest dressing effect of $G_4$, according to $|G_4|^2/d_{41}$ of $\rho_{44}^{(4)}$ in Eq. (5). Figures 3(a2)–3(c2) are the calculation results of Figs. 3(a1)–3(c1) which conform well to the experimental results. Further, with $E_2$ turned on, the triple-dressed signals by scanning $\Delta_4$ at discrete $\Delta_1$ were observed, as shown in Figs. 3(d1)–3(f1). The profile (dashed curve) consisting of baselines in Fig. 3(d1) is the intensity of double-dressed PTS induced by $E_4$ ($E_4^*$) and $E_2$. When we change $\Delta_1$, the intensity of PTS reaches its highest value at $\Delta_2 = -\Delta_1 = -\Delta_3$, because of the term $|G_2|^2/d_{21} + |G_3|^2/d_{30}$ in Eq. (1). In each sub-curve, peaks are the dressed PTS induced by the third-level dressing effect of $G_4$, which meet the condition $\Delta_1 + \Delta_4 = 0$ according to Eq. (1). The peak reaches the smallest value at $\Delta_2 = -\Delta_1$ because of the strongest cascaded interaction between $E_2$ and $E_4$, as depicted by the doubly dressed term $|G_2|^2/d_{21} + |G_4|^2/d_{41}$ in Eq. (1). The total PTS reaches maximum at $\Delta_1 = \Delta_4 = -\Delta_1 = -\Delta_3$. In Fig. 3(e1), the profile (dashed curve) consisting of the baselines shows that the FWM BGS is suppressed by the dressing effect of $E_2$, and it reaches minimum at $\Delta_3 = -\Delta_4$, according to the term $|G_2|^2/d_{21}$ in Eq. (2). Dips in each sub-curve represent that FWM BGS is further suppressed by $E_1$ at $\Delta_1 + \Delta_4 = 0$, according to the term $|G_4|^2/d_{41}$ in Eq. (2). In addition, the suppression dip became shallowest at $\Delta_1 = -\Delta_1$ because of the interplay between $E_2$ and $E_4$. The total FWM BGS reaches its smallest value at $\Delta_4 = -\Delta_2 = -\Delta_1 = -\Delta_3$ because of

![Fig. 3. Measured (a1) PTS, (b1) FWM BGS, and (c1) FL signal versus $\Delta_4$ with $\Delta_1 = 0$ when we block $E_3$ and set $\Delta_1$ at different discrete values. (a2)–(c2) are the calculation results of (a1)–(c1). (d1) PTS, (e1) FWM BGS, and (f1) FL signal versus $\Delta_4$ with $\Delta_1 = \Delta_4 = 0$ at different discrete $\Delta_2$. (d2)–(f2) are the calculation results of (d1)–(f1).](https://example.com/fig3.png)
triple-dressing effect in Eq. (2), corresponding to the strongest PTS. In Fig. 3(f1), the profile peak is FL_R4 signal related to ρ_{44}^{(4)}, and the profile dip is FL_R1 signal dressed by E_2 because of the dressing term |G_3|^2/d_{31} in Eq. (3). The emission peaks in each sub-curve are FL_R4 signals related to ρ_{44}^{(4)}. At the resonance point Δ_2 = −Δ_1, the intensity of emission peak reaches the smallest value because of the term d_{10} + |G_2|^2/d_{21} in Eq. (5). The suppression dips in each sub-curve are dressed FL_R1 signals induced by the dressing term |G_4|^2/d_{41} + |G_2|^2/d_{21}, according to Eq. (3); it becomes deepest at Δ_4 = Δ_2 = −Δ_1, while it gets shallower with changing Δ_2. The corresponding calculations are also present in Fig. 3(d2)–3(f2) which are in agreement with the experimental results.

C. Power Dependence

While scanning Δ_4, the power dependences of the three types of signals were observed in Fig. 4. In Figs. 4(a)–4(c), we show the three types of measured signals from bottom to top with a change in the power of E_4 (P_4) from small to large. In Fig. 4(a), the baselines represent the intensity of the PTS induced by E_2 and E_4. The peaks are enhanced PTS induced by the third-level dressing effect of E_4. With P_4 increasing, the peak becomes higher because of the dressing term |G_4|^2/d_{41} in Eq. (1). In Fig. 4(b), the baselines represent the intensity of FWM BGS dressed by E_2. The dip is FWM BGS suppressed by E_4, according to the term |G_4|^2/d_{41} in Eq. (2). The dip becomes deeper with increasing P_4 because of the enhanced dressing effect. A deeper dip indicates the reflected FWM BGS becomes smaller. In Fig. 4(c), the baselines stand for the FL_R1 signal induced by E_2 and E_4. The dips represent that FL_R1 signals are suppressed by E_4, according to the term |G_4|^2/d_{41} in Eq. (3). The dip becomes deeper with P_4 increasing because of the enhanced dressing effect of E_4. A deeper dip indicates the smaller FL_R1 signal. Peaks within the dips are FL_R4 signals related to ρ_{44}^{(4)}, which become bigger with an increase in the power of generating field E_4.

In contrast, in Figs. 4(d)–4(f) we investigated the three signals by scanning Δ_4 with increasing P_2. The intensity of the dressed PTS decreases from bottom to top in Fig. 4(d) because of the cascade interaction between the dressing field E_2 and E_4, according to the doubly dressed term |G_2|^2/d_{21} + |G_4|^2/d_{41} in ρ_{10}^{(1)} of Eq. (1). The cascade interaction can weaken the dressing effect of E_4 and finally make the PTS peak small. Correspondingly, suppression of the FWM BGS [in Fig. 4(e)], resulting from the dressing effect of E_4, weakens with increasing P_2 also because the cascade dressing field E_2 has an inverse effect compared with E_4. The emission peak of the FL_R4 signal related to ρ_{44}^{(4)} becomes lower with increasing G_2. It results from the suppression caused by the dressing effect of E_2, according to the term d_{10} + |G_2|^2/d_{21} in ρ_{44}^{(4)} of Eq. (5).

D. Phase Dependence

Finally, we regulate the PTS, FWM BGS, and FLS with the relative phase of E_2(Δϕ) by changing the incident angle of E_2 [26]. The experimental results can be obtained by scanning Δ_2 with Δ_1 = 0, as shown in Figs. 5(a)–5(c). With Δϕ considered, Eqs. (1)–(4) can be modified as follows:

\[
ρ_{10}^{(1)} = \frac{iG_1}{d_{10} + |G_3|^2/d_{30} + |G_2|^2 \exp[i(Δϕ + φ_{NL})]/d_{21} + |G_4|^2/d_{41}},
\]

\[
ρ_{10}^{(3)} = \frac{-iG_1G_3G_4}{(d_{10} + |G_3|^2/d_{30} + |G_2|^2 \exp[i(Δϕ + φ_{NL})]/d_{21} + |G_4|^2/d_{41})^2d_{30}},
\]

\[
ρ_{11}^{(2)} = \frac{-|G_1|^2}{Γ_{11}(d_{10} + |G_2|^2 \exp[i(Δϕ + φ_{NL})]/d_{21} + |G_4|^2/d_{41})},
\]

\[
ρ_{22}^{(4)} = \frac{|G_1|^2|G_2|^2}{Γ_{22}d_{10} + |G_2|^2 \exp[i(Δϕ + φ_{NL})]/d_{21})d_{3}},
\]
where $\varphi_{NL}$ is the nonlinear phase of the $E_2$ [27]. It is constant, where $n_2$ is the nonlinear refractive index of the cell related to the density of the $^{85}\text{Rb}$ vapor $N$ (determined by the cell temperature) and $J$ is the intensity of the signals. Thus, it is negligible in the above experiments. With the relative phase $\Delta \phi$ changing from $\pi/2$ to $-\pi/3$, the PTS in Fig. 5(a) can be switched from a dip to a peak. The peaks stand for the transmission enhancement of the probe signal, and the dips stand for the absorption enhancement of the probe signal. During this process, the deepest suppression dip and the highest enhancement peak separately appear at $\Delta \phi = -\pi/6$ ($\varphi_{NL} = \pi/6$, $\Delta \phi + \varphi_{NL} \approx 0$) and $\Delta \phi = 2\pi/3$. In the phase-matching range, where $\Delta \phi$ is altered from $2\pi/3$ to $-\pi/3$, compared with the signals at the reference phase $\Delta \phi = 0$, the FWM BGS [Fig. 5(b)] and FL$_{R2}$ [peaks in Fig. 5(c)] increase when $\Delta \phi < 0$, and become small while $\Delta \phi > 0$ because of the switch of the dressing effect of $E_2$ induced by $\Delta \phi$. With the variation of the dressing effect of $G_2$ caused by $\Delta \phi$, in Figs. 5(b) and 5(c), the deepest suppression dips in FWM BGS and FL$_{R1}$ appear at $\Delta \phi = -\pi/6$ which correspond to the strongest PTS peak. Compared with the signals at the reference phase $\Delta \phi = 0$, the FWM BGS signal and FL$_{R1}$ signal can be suppressed, i.e., the suppression dip become deeper, with $\Delta \phi$ changed as $-\pi/6$. In addition, the FWM BGS FL$_{R2}$ signals can be enhanced, i.e., the suppression dip becomes shallower, with $\Delta \phi$ changed as $\pi/3$ and $2\pi/3$. These variations are because of the switch of the dressing effect caused by the regulation of the relative phase $\Delta \phi$, according to the term $|G_{21}|^2e^{i\Delta \phi}/d_{21}$ in Eqs. (7) and (8). Under the abnormal configuration with $\Delta \phi = \pi$ and $\Delta \phi = -\pi/3$, the suppression dips of FWM BGS and FL$_{R1}$ signals become shallower because the classical effect plays the dominant role.

4. CONCLUSIONS

In summary, we experimentally and theoretically observed the PTS, FWM BGS, and FL signals in a triple-dressed PBG structure, and we can see that the third dressing field can modify the reflection efficiency. In addition, the interplay between two dressing fields is investigated when we change the powers of detunings of incident beams. Moreover, the single-dressed, double-dressed, and triple-dressed PTS, FWM BGS, and FL signals are compared for the first time, to the best of our knowledge, to deeply comprehend the triple-dressing effect on the PBG. Such research could find its applications in optical amplifiers and quantum information processing.

APPENDIX A: DERIVATIONS OF EQS. (1) AND (2) AND THE EQUATIONS ABOUT $R$ AND $T$ IN SECTION 2

A1. Derivation of Eqs. (1) and (2)

Considering the time-dependent Schrödinger equation, using the perturbation chain $\rho^{(3)}_{00} \rightarrow \rho^{(2)}_{10} \rightarrow \rho^{(1)}_{30} \rightarrow \rho^{(1)}_{10}$ (i.e., Liouville pathways with perturbation theory and satisfying phase-matching condition), and rotating wave approximation, we can obtain a series of density matrix equations as follows:

$$\frac{d\rho^{(1)}_{10}}{dt} = -d_{10}\rho^{(1)}_{10} + iG_1 \exp(ik_1 \cdot x)\rho^{(0)}_{30} + iG_2^* \exp(-ik_2 \cdot x)\rho^{(2)}_{20} + iG_4^* \exp(-ik_4 \cdot x)\rho^{(3)}_{40} + \hat{A}G_3 \exp(i\Delta\phi \cdot x) + G_3^* \exp(i\Delta\phi^* \cdot x)\rho^{(1)}_{10},$$

(A1)

$$\frac{d\rho^{(2)}_{30}}{dt} = -d_{30}\rho^{(2)}_{30} + iG_3^* \exp(-ik_3 \cdot x)\rho^{(1)}_{10},$$

(A2)

$$\frac{d\rho^{(3)}_{10}}{dt} = -d_{10}\rho^{(3)}_{10} + iG_3 \exp(ik_3 \cdot x)\rho^{(2)}_{20} + iG_2^* \exp(-ik_2 \cdot x)\rho^{(1)}_{10} + iG_4^* \exp(-ik_4 \cdot x)\rho^{(0)}_{30} + \hat{A}G_3 \exp(i\Delta\phi \cdot x) + G_3^* \exp(i\Delta\phi^* \cdot x)\rho^{(3)}_{40},$$

(A3)

$$\frac{d\rho^{(1)}_{20}}{dt} = -d_{20}\rho^{(1)}_{20} + iG_2 \exp(ik_2 \cdot x)\rho^{(0)}_{10},$$

(A4)

$$\frac{d\rho^{(0)}_{10}}{dt} = -d_{10}\rho^{(0)}_{10} + iG_4 \exp(i\Delta\phi \cdot x)\rho^{(2)}_{40},$$

(A5)

$$\frac{d\rho^{(3)}_{30}}{dt} = -d_{30}\rho^{(3)}_{30} + iG_3^* \exp(-ik_3 \cdot x) + G_3^* \exp(-ik_3^* \cdot x)\rho^{(1)}_{10}.$$

(A6)

By solving Eqs. (A1)–(A6) with the steady-state approximation and the condition $\rho^{(3)}_{00} = 1$ (which is reasonable since the probe field is always weak, compared with other fields), we finally obtain first-order and third-order density matrix elements $\rho^{(1)}_{10}$ and $\rho^{(3)}_{10}$ as Eqs. (1) and (2).

A2. Derivation of the Equations of $R$ and $T$

We differentiate $\partial E_i(x)/\partial x = -\alpha E_i(x) + k \exp(-i\Delta k_\lambda x)E_F(x)$ with respect to $x$ and simplify it by using $-\partial E_F(x)/\partial x = -\alpha E_F(x) + k \exp(i\Delta k_\lambda x)E_i(x)$ to get the following second-order differential equation:

$$\partial^2 E_i(x)/\partial x^2 = -\alpha^2 E_i(x) - k^2 E_i(x) + i\Delta k_\lambda k \exp(-i\Delta k_\lambda x)E_i(x) = 0.$$  

(A7)

After eliminating $E_i(x)$ in $\partial E_i(x)/\partial x = -\alpha E_i(x) + k \exp(-i\Delta k_\lambda x)E_F(x)$ and Eq. (A7), we derive the following equation:
\[ \frac{\partial^2 E_1(x)}{\partial x^2} + \left( -k^2 - \alpha^2 + i\Delta k_x \alpha \right) E_1(x) + i\Delta k_x \partial E_1(x)/\partial x = 0. \] (A8)

The general solution of Eq. (A8) is

\[ E_1(x) = B_1^+ \exp(\lambda_1^+ x) + B_1^- \exp(\lambda_1^- x). \]

Next we substitute the value of \( E_1(x) \) in \( \partial E_1(x)/\partial x = -\alpha E_1(x) + k \exp(-i\Delta k_x)E_F(x) \) to get

\[ E_F(x) = (\lambda_1^+ + \alpha)/k B_1^+ \exp(\lambda_1^+ x) + (\lambda_1^- + \alpha)/k B_1^- \exp(\lambda_1^- x), \]

where \( \lambda_1^+ = -i\Delta k_x/2 \pm [(a - i\Delta k_x/2)^2 + k^2]^{1/2}, \lambda_1^- = \lambda_1^+ + i\Delta k_x. \)

We assume that the length of the \( ^{85}\text{Rb} \) sample is \( d_e \) and apply initial conditions \( E_1(x) = E_0 \) and \( E_1(d_e) = 0 \) to get the values of \( B_1^+ = (\lambda_1^+ + \alpha)/\exp(\lambda_1^+ d_e) \) and \( B_1^- = E_0/\exp(\lambda_1^- d_e) \)

\[ R = |E_1(0)/E_1(0)|^2. \] (A10)

The probe transmission efficiency across the medium with respect to the incident probe field is given as follows:

\[ T = |E_1(d_e)/E_1(0)|^2. \] (A11)

Finally, the reflected FWM BGS (R) and PTS (T) can be obtained.

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