Resolution and Contrast Enhancement of Correlated Imaging in Photon Counting Regime

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We theoretically discuss the four-photon spatial correlation property under different detection schemes. Compared with the two- and three-photon spatial correlation of thermal light source, we found that the enhancement in the visibility and the contrast when the fourth-order spatial correlation function is measured with quadruple coincidence measurement in the single photon counting regime. The resolution, however, is not changed when we scan three of four detectors with identical speed and direction. But if we scan two of three detectors in opposite directions and same speed, simultaneously, other two in identical direction and speed of one detector is twice of that of the other, we can see that both the contrast and resolution are improved significantly. We also show that the conclusion can be applied to the Nth-order ghost imaging.

1. Introduction

Recent studies on the spatial correlated photon pairs of light source brought forth several interesting optical effects, including quantum imaging,1,2) ghost interference,3) quantum lithography,4) and sub-wavelength interference.5) Correlated imaging is a technique to imaging nonlocally of an object, in which two correlated photons are transmitted through a test arm and a reference arm, respectively. By measuring the spatial correlation between the two arms, the image of the object inserted into the test arm can be obtained as a spatial function of the detector position in the reference arm. In 1995, Pittman et al. first demonstrated two-photon imaging experiment based on the quantum entangled of signal and idler photon pairs from spontaneous parametric down conversion (SPDC), and the point-to-point image-forming correlation between the object plane and the image plane resulted from the intrinsic correlation between the signal and idler photons.6) The experiment led to many interesting studies and some debate about whether ghost imaging can be achieved with a classical light source. Abouraddy stated that only quantum entangled sources can be used to realize coincidence imaging and using classical light sources cannot.7) However, by using classical statistical optics, Han theoretically proved correlated imaging with incoherent source,8) and gave a proposal to realize lensless Fourier-transform imaging.9) Then the Gaussian thin lens equations10) and macroscopic differences between the quantum and classical correlated imaging with classical thermal light source were obtained.11) Furthermore, Valencia et al. experimentally demonstrated that the natural, nonfactorizable, point-to-point image-forming correlation is not only the property of entangled photon pairs,12) but the ghost imaging can be realized in the joint detection with chaotic thermal radiation as well.

Correlated imaging with thermal light exhibits many important applications13–15) because of its nonlocal and turbulence free features.16–18) However, it is well known that the visibility of second-order thermal light ghost imaging is always less than 33% for the ghost image formed using thermal light always lies on a large background. Even though the background signal can be subtracted from the total signal, a large number of samplings of signals are required to obtain acceptable quality of the imaging in the thermal ghost imaging setup. This is one of obstacles limiting their practical applications. Some efforts to enhance the visibility have been made, by either cutting the background using electronic band pass filters or increasing the strength of correlation by employing higher order correlations. Agafonov et al. proposed that for coincidence interferometry of three-photon and four-photon,19) the fringe visibility can be much higher than the classical limit of two-photon interference. Besides that, the similarities and differences between third- and second-order correlated imaging for thermal light are theoretically analyzed,20) and experimentally demonstrated.21) Also, a recent work proved that the visibility15) and contrast22) of the ghost diffraction can be improved by using high-order correlation. Nevertheless, a high visibility and contrast does not imply a good image, a high resolution is also needed. The purpose of the present paper is to study the property of high-order thermal ghost imaging by using different detection schemes, and quantitatively show that the visibility, contrast and resolution of the ghost image can be increased with designate detection process. We found that the resolution of fourth-order ghost imaging has no improvement if we simultaneously scan three of four detectors with the same speed and identical direction. However, both the contrast and resolution can be dramatically improved if we scan two detectors in opposite and same speed, simultaneously, another two in identical direction and the speed of one is twice as that of the other. The conclusion can also be applied to the Nth-order ghost imaging.

This paper is organized as follows. In Sect. 2, we discuss the basic theory of thermal light field, including the second-, third-, and fourth-order spatial correlation function. In Sect. 3, we investigate the resolution and contrast properties of third- and fourth-order spatial correlation function under different detection schemes, respectively. Finally, we summarize our results in Sect. 4.
2. Basic Theory

We have theoretically studied the fourth-order spatial correlation properties of thermal light at the single photon counting level, and a theoretical scheme of lensless quantum imaging based on these features is proposed as well.23) A simplified schematic of fourth-order correlated imaging system is shown in Fig. 1, a classical thermal light source, usually created by illuminating a laser beam into a slowly rotating ground glass,24) is divided into four beams, which can be implemented by a combination of three beam splitters. The four beams travel through a test arm (the branch going upward after the first beam splitter) and three reference arms (other three branches), which are characterized by their impulse response functions. An unknown object is inserted into the test arm. To record the intensity distribution at the detection planes with the transverse coordinates, four fiber tips \( p_1, p_2, p_3, \) and \( p_4 \) are set on four planes \( X_1, X_2, X_3, \) and \( X_4, \) which is vertical to the four arms to couple the four beams into four fibers, through which the beams are finally received by four detectors \( D_1, D_2, D_3, \) and \( D_4, \) respectively.

In the theoretical calculations, the fiber tip \( p_1 \) in the object plane \( X_1 \) is fixed and as a function of the spatial shape of the unknown object, and the positions of the fiber tips \( p_2, p_3, \) and \( p_4 \) in the image planes are scanned and the tips positions-dependent quadruple coincidence counting rate recorded. To make the description conform to intuition well, we will describe the fixing and scanning of the positions of fiber tips as fixing and scanning of the positions of detectors in the follows, and the latter description is further simplified as the fixing and scanning of the detectors, whereas no contradiction exists.

In order to simplify the calculation of fourth-order correlation function, the longitudinal distances between the detection planes and the pinhole for the four beams are all same, which can be achieved by designing the adjustable positions of fiber tips. Furthermore, to simplify the calculation into one dimensional, we assume that the four fiber tips \( p_1, p_2, p_3, \) and \( p_4 \) in Fig. 1 have the same \( y \) coordinates in the transverse plane and same polarization direction. In such case, with the Glauber’s correlation functions and the density operator describing fourth-order correlation function of chaotic thermal light,23) we have obtain the normalized fourth-order correlation function as

\[
G^{(4)}(x_1, x_2, x_3, x_4) = 1 + \sin^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) + \sin^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_1) \right) + \sin^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) + \sin^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_2) \right) + \sin^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_3) \right) + \sin^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_4) \right) + \sin^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_4) \right)
\]

\[
+ 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_2) \right) + 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_3) \right)
\]

\[
+ 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_4) \right) + 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_3) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_4) \right)
\]

\[
+ 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_4) \right) + 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_3) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_4) \right)
\]

\[
+ 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_3) \right) + 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_4) \right)
\]

\[
+ 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_4) \right) + 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_3) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_4) \right)
\]

\[
+ 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_3) \right) + 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_4) \right)
\]

\[
+ 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_4) \right) + 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_3) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_4) \right)
\]

\[
+ 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_3) \right) + 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_4) \right)
\]

\[
+ 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_4) \right) + 2 \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_1) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_1 - x_3) \right) \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_4) \right)
\]
respectively. As the three scanning detectors moved together and their transverse coordinates satisfy the relation \(x_3 = 2x_1 - x_2\) and \(x_4 = 2x_2 - x_1\), it is equivalent to measure the correlation function along the direction denoted by green arrows (in the top-left to bottom-right diagonal) shown in (a) and red arrows (in the top-right to bottom-left diagonal) shown in (b), respectively.

\[
\begin{align*}
+ 2 \sin^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) & \sin \left( \frac{\pi \Delta \theta}{\lambda} (x_3 - x_2) \right) \\
+ \cos^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right) \sin^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_4 - x_3) \right) & = 0,
\end{align*}
\]

where \(x_1, x_2, x_3,\) and \(x_4\) are the transverse positions of detectors \(D_1, D_2, D_3,\) and \(D_4,\) respectively, and \(\Delta \theta\) is the angular size of the source viewed from the detectors. Next, if we turn off detector \(D_4\) and scan the two detectors \(D_2\) and \(D_3\) simultaneously with detector \(D_1\) fixed to measure the third-order point-to-spot image-forming correlation function, by removing the terms related to \(x_4\) from Eq. (1). In such case, Eq. (1) is simplified into a normalized third-order correlation function,

\[
G^3(x_1, x_2, x_3) = 1 + 2 \sin^2 \left[ \frac{\Delta \theta(x_2 - x_1)}{2} \right] \sin \left[ \frac{\Delta \theta(x_3 - x_1)}{2} \right] \sin \left[ \frac{\Delta \theta(x_3 - x_2)}{2} \right].
\]

This result can be found in Refs. 20 and 21. Finally, if both detector \(D_1\) and \(D_4\) are removed, we can see that the image forming correlation function of the third-order thermal light shown in Eq. (2) is further simplified into the normalized second-order correlation function

\[
G^2(x_1, x_2) = 1 + \sin^2 \left[ \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right].
\]

This result can be found in Refs. 8, 12, 25, and 26 well.

3. Enhanced Resolution with High-Order Thermal Light

3.1 Fourth-order thermal light

Both the resolution and contrast of the fourth-order ghost image are determined by Eq. (1), and in the following we will study how they change when different detection schemes are adopted. In order to facilitate our discussion, firstly, we define the distance between \(D_1\) and \(D_2\) (\(D_3\)) in transverse \(x\)-coordinates expressed as \(x_{21} = x_2 - x_1\) \((x_{31} = x_3 - x_1)\), and the difference between \(D_2\) and \(D_4\) as \(x_{42} = x_4 - x_2\).

According to Eq. (1), we can numerically simulate the fourth-order correlation function by the three-dimensional plotting and observing this plot from top view as shown in Figs. 2(a) and 2(b). In this picture the magnitude of \(G^4(x_{21}, x_{31})\) is represented by the pseudo-color (or gray level), and \(x_{21}\) \((x_{31})\) in horizontal (vertical) axis is scanned with very small step size as shown in Fig. 2(a), at the same time, the magnitude of \(G^4(x_{21}, x_{42})\) is represented as shown in Fig. 2(b).

In such numerical simulation, the transverse coordinate of object plane \(x_1\) is fixed and that of three other image planes \(x_2, x_3,\) and \(x_4\) are controlled in two different ways, i.e., two different detection schemes are adopted, which correspond to different scanning trajectories shown in Figs. 2(a) and 2(b). In the first detection scheme (FDS), three fiber tips \(p_2, p_3,\) and \(p_4\) are moved with the same direction and speed, so \(x_2 = x_3 = x_4\) is always satisfied in the scanning of the \(x\)-coordinates \(x_2, x_3,\) and \(x_4\) of \(D_1, D_2,\) and \(D_3\). This scheme is equivalent to measuring the fourth-order spatial correlation function along the trajectories indicated by the red arrows.
(from the top-right to bottom-left diagonal) shown in Fig. 2(a) and green arrows (from left to right in the horizontal direction) shown in Fig. 2(b), respectively. In the second detection scheme (SDS), the two fiber tips $p_2$ and $p_3$ are moved with the same speed but opposite directions to make $x_2 = 2x_1 - x_3$ always satisfied, meanwhile, other two fiber tips $p_2$ and $p_4$ are moved with the same direction and the speed of $p_4$ is twice of that of $p_2$, so that the condition $x_4 = 2x_2 - x_1$ is always satisfied, and this is equivalent to measuring along the direction indicated by the green arrows (from the top-left to bottom-right diagonal) shown in Fig. 2(a), and red arrows (from the top-right to bottom-left diagonal) from in Fig. 2(b), respectively.

In the FDS mentioned above with relation $x_2 = x_3 = x_4$, so the fourth-order spatial correlation function shown in Eq. (1) can be simplified into

$$G^{(4)}(x_2) = 6 + 18 \text{sinc}^2 \left[ \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right].$$

Then, in the SDS referred with $x_3 = 2x_1 - x_2$ and $x_4 = 2x_2 - x_1$, the fourth-order point-to-spot correlation function turns into

$$G^{(4)}(x_2) = 1 + 3 \text{sinc}^2 \left[ \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] + 2 \text{sinc}^2 \left[ \frac{2\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] + \text{sinc}^2 \left[ \frac{3\pi \Delta \theta}{\lambda} (x_2 - x_1) \right]$$

$$+ 4 \text{sinc} \left[ \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] \text{sinc} \left[ \frac{2\pi \Delta \theta}{\lambda} (x_2 - x_1) \right]$$

$$+ 4 \text{sinc} \left[ \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] \text{sinc} \left[ \frac{3\pi \Delta \theta}{\lambda} (x_2 - x_1) \right]$$

$$+ 2 \text{sinc} \left[ \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] \text{sinc} \left[ \frac{2\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] \text{sinc} \left[ \frac{3\pi \Delta \theta}{\lambda} (x_2 - x_1) \right]$$

$$+ 2 \text{sinc} \left[ \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] \text{sinc} \left[ \frac{3\pi \Delta \theta}{\lambda} (x_2 - x_1) \right]$$

$$+ 2 \text{sinc} \left[ \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] \text{sinc} \left[ \frac{2\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] \text{sinc} \left[ \frac{3\pi \Delta \theta}{\lambda} (x_2 - x_1) \right]$$

$$+ \text{sinc} \left[ \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right] + \text{sinc} \left[ \frac{2\pi \Delta \theta}{\lambda} (x_2 - x_1) \right].$$

Both of Eqs. (4) and (5) represent the correspondence between the object and image planes in the point-to-spot image-forming process, yet the behaviors of them are different due to the different scanning trajectories shown in Figs. 2(a) and 2(b). As shown in Fig. 3, the fourth-order correlation functions based on Eqs. (4) and (5) are plotted with $x_2$ scanned from $-4$ to $4$ mm. As blue (dot) line shown in Fig. 3, it is obvious that the contrast of correlation peak $G^{(4)}(x_2)$ in Eq. (4) corresponding to the FDS is $\sim 4 : 1$, and the FWHM is to $\sim 0.72$ mm. The FWHM based on Eq. (4) is equal to that of the second-order correlation function in Eq. (3), and means that the resolution of the fourth-order thermal light ghost imaging is equivalent to that of the second-order. However, when the SDS is adopted, the solid (red) line shown in Fig. 3, the contrast of the correlation peak of $G^{(4)}(x_2)$ in Eq. (5) increases to $\sim 24 : 1$ and the FWHM decreased to $\sim 0.33$ mm, about 46% of that in Eq. (4), which means both the resolution and contrast have significant improvement when the SDS is adopted. The narrowing effect of the correlation peak can be attributed to the differences between Eqs. (4) and (5). First, compared with Eq. (4), two kinds of new sinc functions $\text{sinc} \left[ \frac{2\pi \Delta \theta}{\lambda} (x_2 - x_1) \right]$ and $\text{sinc} \left[ \frac{3\pi \Delta \theta}{\lambda} (x_2 - x_1) \right]$ are introduced in Eq. (5), which has narrower width when $x_2$ is scanned. Second, the fifth and sixth terms in Eq. (5) are both product of three sinc functions, and the last six terms are all product of four sinc functions. Such terms can give narrower width and they do not appear in Eq. (4). This comparison means that we can enhance the resolution and contrast in the fourth-order ghost imaging of an object by moving the three scanning detectors $D_2$, $D_3$, and $D_4$ in a way guaranteeing $x_2 = 2x_2 - x_1$ and $x_2 = 2x_1 - x_3$. 3.2 Third-order imaging with thermal light

If we turn off the detector $D_3$ and set the fiber tips of $p_1$ fixed at $x_1 = 0$ in object plane, and move the two fiber tips $p_2$ and $p_3$ to record the triple coincidence counting rate, which will give us the third-order correlation function. Here, there are also two different detection schemes available. In the FDS, $p_2$ and $p_3$ are moved with identical direction and same
speed, so their coordinates always satisfy $x_2 = x_3$, and in the SDS, the two fiber tips $p_2$ and $p_3$ are moved with opposite directions, but identical speed to make their coordinates satisfy $x_2 = 2x_1 - x_3$. So in the FDS, the relation $x_2 = x_3$ makes the third-order spatial correlation function in Eq. (2) reduced to

$$G^{(3)}(x_1, x_2, x_3) = 2 + 4 \text{sinc}^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right). \quad (6)$$

The three kinds of different order correlation functions with FDS shown in Eqs. (4), (6), and (3) are plotted in Fig. 4, in which the blue (dot-dash) with maximum value 24; minimum value 6, green (dot) with maximum value 6, minimum value 2, and black (solid) with maximum value 2; minimum value 1, correlation peaks are the results of the fourth-, third-, and second-order correlation functions, respectively. The three functions are all obtained under the same speed and opposite directions. This is equivalent to scanning detectors moved together and transverse coordinates satisfy the relation $x_1 = -x_2$. When $D_2$ and $D_3$ are scanned with the same speed and direction, the FWHM with $x_2$ varying is determined by the term $\text{sinc}^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right)$. When $D_2$ and $D_3$ are scanned with same speed and opposite directions, the FWHM is narrower than that of the correlation function in Eq. (6) for two reasons. First, the term $\text{sinc}^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right)$ in Eq. (7) has narrower FWHM than $\text{sinc}^2 \left( \frac{2\pi \Delta \theta}{\lambda} (x_2 - x_1) \right)$ in Eq. (6). Second, the last term of Eq. (7) is product of three sinc functions, with no counterpart in Eq. (6), which can further make the FWHM of the correlation function in Eq. (7) narrower.

Finally, to seek for the contrast, visibility and resolution varied with different order of correlation functions in the SDS, we plot the correlation functions of third- and fourth-order according to Eqs. (5) and (7), as shown in Fig. 5(b). In the plots, blue (with maximum value 24; minimum value 1) correlation peak with contrast 3 : 1 corresponds to the Eq. (6), while the red one (with maximum value 6; minimum value 1) with contrast 6 : 1 to the Eq. (7). The FWHM of the red correlation peak is $\sim 0.48$ mm, which is $\sim 67\%$ of that of the blue correlation peak. Such results mean that both the contrast and resolution of the third-order imaging are improved with the SDS is utilized. Such improvement can be explained by comparing Eqs. (6) and (7) as discussed with fourth-order imaging. When the two detectors $D_2$ and $D_3$ are scanned with the same speed and direction, the FWHM with $x_2$ varying is determined by the term $\text{sinc}^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right)$. When $D_2$ and $D_3$ are scanned with same speed and opposite directions, the FWHM is narrower than that of the correlation function in Eq. (6) for two reasons. First, the term $\text{sinc}^2 \left( \frac{2\pi \Delta \theta}{\lambda} (x_2 - x_1) \right)$ in Eq. (7) has narrower FWHM than $\text{sinc}^2 \left( \frac{\pi \Delta \theta}{\lambda} (x_2 - x_1) \right)$ in Eq. (6). Second, the last term of Eq. (7) is product of three sinc functions, with no counterpart in Eq. (6), which can further make the FWHM of the correlation function in Eq. (7) narrower.

4. Conclusions

In summary, we have theoretically investigated the relation among the second-, third-, and fourth-order spatial correlation property of thermal light. Based on this relationship, we focused on the spatial correlation functions of third- and fourth-order thermal light under two different detection schemes. This is equivalent to measuring the ghost image utilized with the FDS. When the order of correlation increases from 2, to 3, and to 4, the resolution and contrast are improved with the SDS.

Fig. 4. (Color online) Blue (dot-dash), green (dot), and black (solid) correlation function peaks are fourth-, third-, and second-order spatial correlation function from Eqs. (4), (6), and (3), respectively. The FWHM of blue, green, and black correlation peaks are all same.

Fig. 5. (Color online) (a) Blue (dot) and red (solid) peaks are from Eqs. (6) and (7), respectively. The FWHM of blue (the third-order, two scanning detectors move in opposite directions and transverse coordinates satisfy the relation $x_1 = -x_2$) is $\sim 67\%$ of the red (the third-order, two scanning detectors moved together and transverse coordinates satisfy the relation $x_2 = 2x_1 - x_3$), which means the higher resolution. (b) Blue (dot) and red (solid) correlation function peaks are from Eqs. (5) and (7), respectively.
schemes. In the FDS with three detectors scanned with same speed and identical direction, we have found that the visibility and contrast are effectively improved in the four-photon thermal light ghost imaging compared with those in the two- and three-photon imaging, but the imaging resolution is not improved. A significant resolution enhancement in fourth-order ghost imaging has been demonstrated if the SDS is utilized, in which two detectors are scanned in opposite directions and same speed, meanwhile, two detectors are scanned with identical directions, in which the scanning speed of one detector is twice of that of another one. In addition, the results can be generalized to Nth-order coherence of chaotic thermal light which has the potential in giving higher contrast (\(\sim N:1\)), visibility (\(\sim(N! - 1)/(N! + 1)\)) and resolution (\(\sim 1/\sqrt{N}\)). Such scheme of can be experimentally demonstrated by generalize experiment system for second-order ghost imaging, and if experimentally demonstrated, it can have interesting applications to more precise imaging and measurement.

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